Unemployment in a Production Network

Finn Schüle and Haoyu Sheng

November 26, 2024

Abstract

We model a production network economy with sectoral and occupational unemployment by incorporating matching between job seekers across various occupations and employers in different production sectors. We derive expressions that unpack how the impact of microeconomic shocks on output and unemployment depends on the interaction between the network linkages, search costs, and changes in labor market tightness. When labor markets are slack, our model predicts larger output and employment responses because the network-adjusted labor productivity gain outweighs search costs. Calibrating our model to the U.S. economy, we demonstrate that our model significantly amplifies the response of aggregate output and unemployment to productivity shocks in any sector and changes the relative importance of sectors to aggregate output and unemployment. Our model nearly doubles the output response compared to an efficient production network and triples the unemployment response compared to a multi-sector search model following a productivity shock to durable manufacturing.

JEL Codes: E1, J3, J6

We thank Gabriel Chodorow-Reich, Fernando Duarte, Gauti Eggertsson, Alexandre Gaillard, Amy Handlan, Şebnem Kalemli-Özcan, Yann Koby, and Pascal Michaillat for helpful comments and advice; Rosemary Kaiser for her insightful discussion; participants at Brown Macro Breakfast, WEAI, and Midwest Macro for their questions and comments; and seminar speakers at Brown University for their invaluable feedback. All errors are our own. We thank the Orlando Bravo Center for Economic Research and the Brown Economics Department for financial support.

1. Introduction

Modern economies feature intricate production networks and specialized labor markets. The interplay between output and unemployment across sectors calls for an integrated modeling approach that combines production networks with search frictions. Incorporating search frictions allows us to study unemployment, an economic force important for both individual households (Jacobson et al. 1993; Bertheau et al. 2023) and society at large (Borgschulte and Martorell 2018; Mas and Pallais 2019; Michaillat and Saez 2021), while integrating sectoral linkages enables us to examine cross-sectoral spillovers in output and employment. A model that has both ingredients sheds light on a crucial question: which jobs and sectors are most affected by sector-specific shocks?

In this paper, we introduce a novel theoretical framework to answer this question. Our approach hinges on three key components. First, our model features a production network using a similar setup to Baqaee and Farhi (2020), mirroring the intricate interdependencies within any modern economy, as highlighted by recent supply chain disruptions during the COVID-19 pandemic. Second, it includes workers from multiple occupations, ranging from specific job roles like engineers to location-specific positions like Detroit-based maintenance and repair workers. This flexible definition of labor markets helps us examine the impact of sectoral shocks on different worker groups. Third, the model features unemployment. Unemployment exists because firms must search for suitable workers to fill job openings, leading to a matching process that is inherently time-consuming. With these realistic economic features, productivity shock impacts sectoral output directly through productivity and intermediate goods production, and indirectly through labor market conditions.

The model reveals a novel channel through which shocks propagate, influenced by the interplay between costly job searches and production linkages, affecting both sector-specific and aggregate output and unemployment levels. Think about, for instance, the interconnectedness of automobile manufacturers and transportation firms, linked through product sales and the specific types of labor they use. A positive productivity shock to car manufacturing, leading to cheaper vehicles for the transportation sector, can reduce transportation service costs for car manufacturers. This leads to an expansion in output in both sectors, increasing the demand for maintenance and repair mechanics and thus affecting employment. Conversely, a tighter labor market can increase hiring costs for mechanics, negatively influencing sectoral productive capacity. We derive tractable expressions, which consist of network linkages, search costs, and labor market tightness, for how technology shocks disseminate and aggregate across the production network in the presence of search frictions that capture exactly this intuition.

Specifically, we show that the aggregate impact on output and unemployment comprises two components. The first component mirrors Hulten (1978)'s foundational aggregation theorem, where the aggregate output response is a sales-share-weighted sum of sectoral technology shocks. The second term, which we call the search-and-matching channel, captures complex interactions between the production and labor market structures. For example, this includes how a positive shock to a car manufacturer impacts the labor demand for mechanics from a downstream transportation firm and how the resulting labor market for mechanics further alters production decision until equilibrium is restored. We recover Hulten's theorem as an edge case in our model: when relative wages adjust exactly according to the occupational-labor-share-weighted marginal product of labor, tightness—the number of vacancies over the number of unemployed workers—remains unchanged, effectively eliminating all additional propagation coming from labor market matching frictions.

The extent to which our novel propagation mechanism amplifies shocks depends on two factors: the difference between network-adjusted productivity gains and wage adjustments, and labor market tightness. As demonstrated by Shimer (2005) and Hall (2005), the search-and-matching setup admits a range of assumptions about wages, each with very different quantitative consequences. If wages are somewhat rigid, meaning they adjust less than the network-adjusted productivity gains from positive shocks, it becomes beneficial for firms to hire more production workers. Our analysis illustrates that canonical rigid wage assumptions, including flexible wages in models without production networks,[1] satisfy this condition. However, hiring more production workers requires firms to allocate some of their workforce to recruiting. In tight labor markets, the rising costs of recruitment can offset the benefits of enhanced worker productivity, and when these costs are sufficiently high, the search channel can dampen the shock propagation.

In addition to technology shocks, our setup allows us to explore another set of shocks: shocks to the size of the labor force. We can think of these shocks as coming from an underlying model of occupational choice, as in Humlum (2021), or from an underlying model of migration, as in Fernandez-Villaverde (2020). Unlike in the standard produc-

[1] This is because network adjustments can amplify productivity gains and losses, so the network-adjusted productivity can change more than one-for-one.

tion networks setup, where a positive shock to the size of the labor force mechanically increases output due to firms being forced to hire all available workers, our framework allows the firms' hiring decision to vary depending on wages. For instance, if wages remain relatively stable following a positive labor force shock, firms may not significantly increase their hiring, dampening the overall effect on output while generating large changes to unemployment.

We calibrate our model to U.S. data to quantify the empirical relevance of the theoretical channels outlined above. We use sectoral vacancy and hiring data from the Job Openings and Labor Turnover Survey (JOLTS), sectoral unemployment data from the Current Population Survey (CPS), and occupation usage data from the Bureau of Labor Statistics' (BLS) Occupational Employment and Wage Statistics (OEWS) to estimate matching parameters and baseline tightness. Industry sales shares and factor usage from the U.S Bureau of Economic Analysis (BEA) map directly into production function elasticities in our Cobb-Douglas setup. We then introduce productivity shocks to each sector to demonstrate the key role of both the production network and the search-and-matching structure of the labor market. Accounting for the production network amplifies productivity effects and leads to much broader impacts on unemployment compared to a multi-sector model without production linkages. Including search and matching in the model introduces unemployment and, under realistic assumptions about wage changes, further enhances these effects beyond what is observed in models with only production linkages and no search and matching.

Assuming the real wage is partially rigid[2], aggregate output and unemployment respond significantly more to productivity shocks in each sector in our model than in a model featuring only production linkages or only search-and-matching frictions. On top of increasing the magnitude of responses, accounting for both network linkages and search-and-matching changes the relative importance of each sector for aggregate output and unemployment. For instance, a pure production networks model attributes too little importance to the retail and wholesale trade sector but too much importance to the non-durable manufacturing sector. A multi-sector search-and-matching model attributes too much importance the education and health sector, but too little to information services. In our full model, shocks to the professional and business service

[2]We define the real wage as the nominal wage over the network-adjusted price. We assume that a 1% increase in the network-adjusted marginal product of labor results in a 0.7% increase in real wages, based on the based on the elasticity estimate of total earnings for job movers in relation to productivity. This estimate comes from Haefke et al. (2013), who analyze panel data tracking a sample of production and supervisory workers from 1984 to 2006, finding an elasticity of 0.7.

sector have the largest effect on aggregate output. This sector ranks fourth in a pure production network and sixth in a multi-sector search-and-matching model without production linkages.

These differences between our model and the two baselines arise because our network-adjusted search-and-matching channel interacts the network significance of a firm with its employment share in the occupational labor markets. A search model captures that larger employers are more important for output and unemployment, but not that sectors which sell to large employers are also important through the labor channel. A production networks model can capture that sectors which are major suppliers to other sectors are more important, but not how this translates to the labor productivity of workers and that each sectors employment share therefore matters beyond the direct effect on intermediate inputs. Our model captures both features and suggests a that a sectors labor-adjust network centrality is an important determinant of the output and unemployment responses to sector level shocks.

Continuing with our illustrative example, we apply a positive productivity shock to the durable manufacturing sector and find that the differences between our model and our baselines persist for the sector level response of output and the occupation level response of unemployment. A 1% positive technology shock to the durable manufacturing sector increases the transportation sector's output by 0.2% and decreases the unemployment rate for maintenance and repair workers by 0.3 percentage points. In addition, aggregate output increases by 0.4%, and the aggregate unemployment rate decreases by 0.2 percentage points. Without search-and-matching frictions, the same shock would increase transportation output by only a quarter as much, increase aggregate output by just over half as much, and would not affect unemployment. On the other hand, a model with only search frictions would predict under half of the increase in aggregate output and just a quarter of the decrease in unemployment.

This paper contributes to the production network literature. Since early contributions by Long and Plosser (1983); Acemoglu et al. (2012); Jones (2011), a recent literature has incorporated inefficiencies into production network models, including markups (Liu 2019; Baqaee and Farhi 2020), financial frictions (Bigio and La'O 2020), and nominal rigidities (Rubbo 2023; La'O and Tahbaz-Salehi 2022; **?**; Baqaee and Farhi 2022; di Giovanni et al. 2023). While these models provide a more realistic view of product and financial market inefficiencies, their treatment of labor markets does not allow for unemployment. Incorporating search and matching allows us to derive closed-form expressions for the first-order propagation of sector-level shocks to output *and* unemployment that are analogous to propagation results presented in this production network literature (e.g., Baqaee and Farhi 2020).

Similar to Baqaee and Farhi (2022) and di Giovanni et al. (2023), who examine the impact of labor supply shocks in New Keynesian production network models, our model features endogenous wedges operating through labor market frictions. In Baqaee and Farhi (2022) and di Giovanni et al. (2023), Keynesian unemployment arises due to labor demand failing to meet inelastic labor supply when wages are downwardly rigid. By modeling search-and-matching between sectoral employers and occupational employees, we provide important microfoundations for unemployment and labor market frictions in production network models. Our definition of unemployment is more comprehensive than Keynesian unemployment, which only occurs in demand-constrained sectors during economic downturns. In addition, we do not assume that occupations are sector specific. Workers can therefore flow between sectors because multiple sectors employ workers from the same occupation. Sector specific segmentation arises from underlying segmentation at the occupation level and from search and matching frictions. We therefore provide important microfoundations for sector level segmentation which di Giovanni et al. (2023) show is a key driver of recent inflation.

This paper also fits in the longstanding literature on search and matching in labor markets, pioneered by Pissarides (1984), Pissarides (1985), Diamond (1982a), Diamond (1982b), Mortensen (1982a), and Mortensen (1982b). Our particular labor market specification closely aligns with Michaillat and Saez (2015) and Landais et al. (2018). Like them, we frame search costs in terms of the recruiter-producer ratio and work primarily within a static setup for tractability. While the textbook model of Pissarides (2017) generates qualitatively reasonable labor market responses to shocks, Hall (2005) and Shimer (2005) highlight that the magnitude of observed fluctuations for key labor market characteristics is too large to be quantitatively explained by shocking the textbook search-and-matching model. Hall (2005) proposes including rigid wages,[3] which we find also increases the magnitude of the output and unemployment response in our network context. In our paper, the network adjustments on labor productivity provide an additional amplification channel for how aggregate outcomes respond to productivity shocks. The marginal product of labor in our model is network adjusted and more sensitive to productivity shocks than that in a network-less economy. We therefore need less wage rigidity than in a network-less model.

The rest of the paper is organized as follows. Section 2 describes our model and

[3] See Bewley (2005) for a survey of empirical evidence.

defines the equilibrium. Section 3 derives expressions for first-order changes in output and employment in response to changes in technology and the labor force size. Section 4 describes the data used to calibrate the model and presents illustrative examples to demonstrate the quantitative importance of our theoretical channels. Section 5 concludes.

2. Model

Our model is a static multi-sector production network model, as in Baqaee and Farhi (2020) and Bigio and La'O (2020), and features J production sectors, indexed by i. However, it deviates from these production network models in the labor market structure, with $\mathcal{O}$ occupations indexed by o. Production requires both intermediate inputs and labor: workers in each occupation need to search for jobs and firms must hire workers. For simplicity, we assume that workers do not transition from one occupation to another when they become unemployed and instead search for a job within the same occupation. In this section, we present the model and characterize its equilibrium. For expositional clarity, the model features Cobb-Douglas production functions and preferences. We derive results for general constant returns-to-scale technology in the Online Appendix.

2.1. Households

Preferences. A representative household with homothetic preferences over the goods from each sector chooses a consumption bundle to maximize utility

$$\mathcal{U}(\{c_i\}_{i=1}^{J}) = \sum_{i=1}^{J} \sigma_i \log c_i,$$

where

$$\sum_i \sigma_i = 1.$$

Budget constraint. The representative household inelastically supplies a labor force of size H_o to occupation o, and pools all income. Its budget constraint is

$$\sum_{i=1}^{J} p_i c_i = \sum_{o=1}^{\mathcal{O}} w_o L_o + \sum_{i=1}^{J} \pi_i,$$

where c_i is the final consumption of sector i's output, p_i is the price of sector i's good, w_o is the wage in occupation o, L_o is the labor used in sector o, and π_i are sector i's profits.

Optimization. The household faces the following optimization problem:

$$\max_{\{c_i\}_{i=1}^{J}} \mathcal{U}(\{c_i\}_{i=1}^{J}),$$

subject to

$$\sum_{i=1}^{J} p_i c_i = \sum_{o=1}^{\mathcal{O}} w_o L_o + \sum_{i=1}^{J} \pi_j.$$

The consumption choices satisfy the first-order condition

$$\sigma_i = \frac{p_i c_i}{\sum_{j=1}^{J} p_j c_j}. \tag{1}$$

2.2. Sectors

Production. Each sector is populated by a representative firm. The firm in sector i employs workers in occupation o, denoted as N_{io}, and uses intermediate inputs from sector j, represented by x_{ij}, to produce output y_i using constant returns Cobb-Douglas production functions. For simplicity, we directly refer to the representative firm as a sector.

$$y_i = A_i \Pi_{j=1}^{J} \Pi_{o=1}^{\mathcal{O}} x_{ij}^{\alpha_{ij}} N_{io}^{\beta_{io}}, \tag{2}$$

where

$$\sum_{j=1}^{J} \alpha_{ij} + \sum_{o=1}^{\mathcal{O}} \beta_{io} = 1.$$

The goods produced by a sector can be used as intermediate inputs by other sectors or can be consumed by the household.

Profits. Production costs include labor cost, intermediate input cost, and fixed factor cost. The profit π_i for sector i is

$$\pi_i = p_i y_i - \sum_{o=1}^{\mathcal{O}} w_o L_{io} - \sum_{j=1}^{J} p_j x_{ij},$$

where

$$L_{io} = (1 + \tau_o(\theta_o))N_{io}.$$

Here, L_{io} denotes the total number of workers from occupation o hired by sector i, N_{io} denotes the number of productive occupation o workers, and $\tau_o = \frac{L_{io}-N_{io}}{N_{io}}$ is the recruiter-producer ratio. As we explain in greater detail in Section 2.3, we assume that to generate hires, some of the occupation o workers in sector i must work in recruiting. This drives a wedge between total employment and productive employment, which we call the recruiter-producer ratio.

Optimization. We assume that sectors are price takers in both input and output markets. Sectors choose $\{N_{io}\}_{o=1}^{\mathcal{O}}$ and $\left\{x_{ij}\right\}_{j=1}^{J}$ to maximize profits:

$$\max_{\{N_{io}\}_{o=1}^{\mathcal{O}}, \left\{x_{ij}\right\}_{j=1}^{J}} \pi_i \left(\{N_{io}\}_{o=1}^{\mathcal{O}}, \left\{x_{ij}\right\}_{j=1}^{J} \right).$$

The profit maximization problem implies that firms choose inputs so that the expenditure shares match the production elasticities:

$$\alpha_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \tag{3}$$

$$\beta_{io} = (1 + \tau_o(\theta_o)) \frac{w_o N_{io}}{p_i y_i}. \tag{4}$$

2.3. Labor Markets

We assume there are $\mathcal{O}$ occupations with separate labor markets. Occupation o has a labor force of H_o possible workers. This is a static, one-shot economy, where all workers start out unemployed and must search for jobs. Similarly, to hire workers, firms must post vacancies, which costs labor to maintain. The exogenous recruiting cost,

r_o, measures the units of labor required for a firm to maintain each posted vacancy in occupation o. We assume that recruiters come from the same pool of workers and receive the same wage as the employees they are tasked with hiring. In other words, occupation o workers hire other occupation o workers.

Matching functions. Hires are generated by a constant returns Cobb-Douglas matching function. The number of matches depends on the number of workers searching for a job in occupation o, H_o, and the number of vacancies posted in occupation o, V_o:

$$h_o = \min\left\{\phi_o H_o^{\eta_o} V_o^{1-\eta_o}, H_o, V_o\right\},$$

where V_o is the sum of sectoral vacancy postings v_{io} and occupational labor market tightness is $\theta_o = \frac{V_o}{H_o}$.[4] Below, we assume that h_o always equals $\phi_o H_o^{\eta_o} V_o^{1-\eta_o}$ and check that this condition is not violated in our calibration.

The vacancy-filling rate, $\mathcal{Q}_o$, is the fraction of vacancies that end up being filled. Similarly, the job-finding rate, $\mathcal{F}_o$, is the fraction of workers who end up finding a job. Both rates can be expressed as functions of tightness:

$$\mathcal{Q}_o(\theta_o) = \frac{h_o}{V_o} = \phi_o \theta_o^{-\eta_o}, \quad \mathcal{F}_o(\theta_o) = \frac{h_o}{U_o} = \phi_o \theta_o^{1-\eta_o}.$$

Tightness therefore plays a crucial role in determining how costly it is for firms to hire workers. In a tight labor market, the vacancy-filling rate decreases, meaning that firms must post more vacancies for a given level of desired employment, thereby increasing their hiring costs.

Labor supply. A fraction $\mathcal{F}_o(\theta_o)$ of the labor force finds a job and is employed at the end of the period. We call this the labor supply, and it satisfies

$$L_o^s(\theta_o) = \mathcal{F}_o(\theta_o) H_o. \tag{5}$$

[4]Because our model is static, and all workers in the labor force must search for a job within the single period, H_o plays the same role in our setup as U_o. However, we find similar expressions for labor demand and labor supply as a function of θ, if we instead define $\theta_o = \frac{V_o}{U_o}$, a stock of existing employed workers, and assume balanced labor market flows.

Labor demand. We assume that firms take occupation-level tightness as given.[5] Suppose sector i wants to employ N_{io} productive workers from occupation o. To hire N_{io} productive employees, it must post $v_{io} = \frac{N_{io}}{\mathcal{Q}_o(\theta_o)-r_o}$ vacancies. Since posting each vacancy requires r_o recruiters, once we account for the number of required recruiters, total occupation o employment in sector i is $L^d_{io} = N_{io} + r_o v_{io}$.

Conveniently, we can relate productive employment to total employment with the recruiter-producer ratio (the ratio of recruiters to productive workers),[6] which does not depend on any sector-specific terms:

$$\tau_o(\theta_o) \equiv \frac{L^d_{io} - N_{io}}{N_{io}} = \frac{r_o}{\mathcal{Q}_o(\theta_o) - r_o}.$$

Using this expression for the recruiter-producer ratio, total labor demand by sector i for occupation o is

$$L^d_{io} = (1 + \tau_o(\theta_o))N_{io},$$

where N_{io} is determined by the sectors' profit maximization problem. In the language of the production network literature, τ_o acts as an endogenous wedge on sectors' labor costs. This wedge plays an important role in how shocks propagate through the production network through labor demand.

Finally, we define aggregate occupation o labor demand as the sum of sectoral labor demands:

$$L^d_o(\theta_o) = \sum_{i=1}^{J} L^d_{io}(\theta_o) = \sum_{i=1}^{J}(1 + \tau_o(\theta_o))N_{io}. \tag{6}$$

Wages. In matching models, workers and firms meet in a situation of bilateral monopoly. The resulting mutual gains from trade mean that wages are not determined by the model's equilibrium conditions[7] and must instead be pinned down by some wage-setting norm chosen by the researcher. For now, we simply assume that wages are a

[5] We can think of each sector as being populated by many identical competitive firms so that each firm only has an infinitesimal impact on aggregate vacancies and therefore on aggregate tightness.

[6] We follow Michaillat and Saez (2015) and use the notion of the recruiter-producer ratio. Introducing recruiters frames the recruiting costs explicitly as the number of workers, or the fraction of each worker's time, dedicated to recruiting, which we deem realistic.

[7] Wages are only constrained to fall within a range where both workers and firms benefit from the match. However, this range can be wide since workers usually have a strong preference for employment over unemployment, and for firms, the process of finding a new match is costly.

function of the underlying shocks in our model:

$$w_o = g_o\left(\{A_i\}_{i=1}^{J}, \{H_o\}_{o=1}^{\mathcal{O}}\right). \tag{7}$$

Since the underlying shocks in our model are to sectoral productivity and occupational labor force sizes, this is not a restrictive assumption. It nests both nominally rigid and real rigid wages, revenue sharing between workers and firms, and fully flexible wages. For example, nominal wages can be a function of productivity as in Blanchard and Galí (2010) and Michaillat (2012), or it can be constant.

2.4. Equilibrium

Given exogenous variables $\left\{\{A_i\}_{i=1}^{J}, \{H_o\}_{o=1}^{\mathcal{O}}\right\}$ and wages $w_o = g_o\left(\{A_i\}_{i=1}^{J}, \{H_o\}_{o=1}^{\mathcal{O}}\right)$ for each o, an equilibrium is a collection of allocations $\{\{y_i\}_{i=1}^{J}, \{x_{ij}\}_{j=1}^{J}, c_i, \{N_{io}\}_{o=1}^{\mathcal{O}}, \{\theta_o\}_{o=1}^{\mathcal{O}}\}$ and prices $\{p_i\}_{i=1}^{J}$ such that

(i) the allocations solve the household's problem (Equation 1),

(ii) the allocations solve the firm's problem (Equations 2–4),

(iii) goods markets clear

$$y_i = c_i + \sum_{j=1}^{J} x_{ij} \qquad \forall i \in \{1, 2, \ldots, J\}, \tag{8}$$

(iv) labor markets, specified by Equations 5 and 6, are in equilibrium

$$L_o^d = L_o^s \qquad \forall o \in \{1, 2, \ldots, \mathcal{O}\}, \tag{9}$$

(v) and wages are set according to Equation 7.

3. Theoretical Results: The Propagation, Aggregation, and Amplification of Shocks

In this section, we describe our main theoretical results. Shocks are defined as small proportional changes in the exogenous variables. We first derive the propagation of technology and labor force shocks at the disaggregated occupation and sector level.

We then compute the first-order responses of output and unemployment, which are the main endogenous variables of interest in the model. Subsequently, we present our aggregation theorem for idiosyncratic shocks. In the Online Appendix, we show how to generalize the results to any constant returns production function in the case of one occupation per sector.

Our main results highlight the theoretical importance of modeling search and matching in the labor market and production linkages in conjunction. Labor demand for a particular occupation depends not only on that occupation's importance to each sector but also on the production linkages between sectors. Labor demand in turn affects tightness in the labor market, creating an endogenous matching wedge that affects the propagation of shocks. We find that for a knife edge case where wage changes are exactly proportional to changes in the marginal product of labor, labor market tightness is unaffected by shocks to productivity or labor supply. As a result, our model behaves as if there were no search-and-matching frictions in the labor market. However, when this knife edge case does not hold, search and matching changes the sectoral, occupational, and aggregate effects of shocks. We find that for plausible parameter values, based on existing estimates of the recruiter-producer ratio and wage rigidity, search-and-matching frictions amplify the effects of sector- or occupation-specific shocks.

3.1. The Propagation of Shocks

We are interested in how two sets of endogenous variables—sector-level output and occupation-level unemployment—change in response to changes in technology and the labor force size. After determining how sector- and occupation-level output and unemployment change, we show how to aggregate using our model's structure.

Our results are entirely first order, and for convenience, we express the first-order relationship between relative wages and shocks as follows:

$$d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = \boldsymbol{\Lambda_A} d\log \boldsymbol{A} + \boldsymbol{\Lambda_H} d\log \boldsymbol{H},$$

where $d\log \boldsymbol{w}$ and $d\log \boldsymbol{H}$ are $\mathcal{O} \times 1$ dimensional vectors capturing first-order changes in wages and the labor force size. $d\log \boldsymbol{p}$ and $d\log \boldsymbol{A}$ are $J \times 1$ dimensional vectors capturing first-order changes in prices and productivity. $\mathcal{L}$ is the occupational-share matrix, a $\mathcal{O} \times J$ matrix with the share of occupation o workers employed in each sector along the rows. The $(o, j)^{\text{th}}$ entry is $\frac{l_{jo}}{L_o}$. This matrix captures each sector's importance as an employer of occupation o workers. $\boldsymbol{\Lambda}_A$ and $\boldsymbol{\Lambda}_H$ are $\mathcal{O} \times J$ and $\mathcal{O} \times \mathcal{O}$ coefficient matrices that capture how wages respond to technology and labor force shocks to first order. For instance, the $(o, i)^{\text{th}}$ entry of $\mathbf{\Lambda}_A$ captures how wages in occupation o respond to technology shocks in sector i.

We write the first-order approximation in terms of the relative wage—the difference between the change in nominal occupational wages and the change in occupation-share-weighted prices—because this is the effective wage that determines labor demand for workers in each occupation. As we show below, changes in relative prices are themselves determined by changes in exogenous variables. Working with price-adjusted wages is therefore an algebraically convenient but innocuous choice.

The following propositions describe how labor market tightness, output, and unemployment respond to shocks (for a detailed derivation, see Appendix A.1). We start with first-order changes in tightness, $d\log\theta$, which is an important endogenous variable in matching models that drives how search costs change as labor demand and supply change. Several recent papers also demonstrate that tightness was a better predictor of the state of the labor market post-COVID than more standard measures of labor market slack (Benigno and Eggertsson 2023; **?**). Therefore, how tightness changes is of independent interest, and understanding these changes will also play a key role for output and unemployment.

PROPOSITION 1. *Let* $\mathbf{\Psi} = (\boldsymbol{I} - \mathbf{\Omega})^{-1}$ *denote the Leontief inverse,*[8] *where* $\mathbf{\Omega}$ *is the* $J \times J$ *input-output matrix. Let* $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}$ *denote the* $J \times \mathcal{O}$ *matrix of production elasticities to labor inputs, and let* $\mathcal{M}$ *be a diagonal matrix with the* $\mathcal{O}$ *matching elasticities along the main diagonal. Moreover, let* $\mathcal{T}$ *be a diagonal matrix with the* $\mathcal{O}$ *recruiter-producer ratios along the main diagonal.*

Given occupational labor force shocks $d\log\boldsymbol{H} = \left[d\log H_1, \cdots, d\log H_{\mathcal{O}}\right]'$ *and sectoral productivity shocks* $d\log\boldsymbol{A} = \left[d\log A_1, \cdots, d\log A_J\right]'$*, the first-order responses of labor market tightness* $d\log\theta = \left[d\log\theta_1, \cdots, d\log\theta_{\mathcal{O}}\right]'$ *follows*

$$d\log\theta = \Pi_{\theta,A}\, d\log\boldsymbol{A} + \Pi_{\theta,H}\, d\log\boldsymbol{H},$$

where

$$\Pi_{\theta,A} = \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}(\mathcal{L}\mathbf{\Psi} - \mathbf{\Lambda}_A),$$
$$\Pi_{\theta,H} = \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}\left(\mathcal{L}\mathbf{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} - \boldsymbol{I} - \mathbf{\Lambda}_H\right),$$

[8] The Leontief inverse captures each sector's importance as a direct and indirect supplier to every other sector.

$$\Xi_\theta = \mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left[\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right].$$

The equilibrium response in tightness is jointly determined by changes in labor supply and labor demand. Labor supply increases when the labor force size rises or when the job-finding rate rises. Labor demand increases when the vacancy-filling rate rises, wages fall relative to prices, or productivity rises.

Intuitively, a productivity shock affects labor demand by directly changing a sector's productive capacity. A productivity shock also impacts prices and output and therefore indirectly changes labor usage in other sectors through production linkages. Algebraically, the difference between the occupation-share-adjusted Leontief inverse $\mathcal{L}\boldsymbol{\Psi}$ and the relative wage coefficients $\boldsymbol{\Lambda}_A$ captures the positive net effect that an exogenous shock has on labor demand. Similarly, $\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{\Lambda}_H$ captures the net effect of shocks to the size of the labor force. The multiplicative constant $\left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}$ captures how the changes in labor demand cascade through the network, accounting for the effects of search and matching. This term therefore plays the role of a Leontief inverse in our expression for first-order changes in tightness.

With our expression for changes in tightness in hand, we can now derive first-order changes in output across the network in Proposition 2.

PROPOSITION 2. *The first-order response of sectoral output* $d\log\boldsymbol{y} = \left[d\log y_1, \cdots, d\log y_J\right]'$ *follows*

$$d\log\boldsymbol{y} = \boldsymbol{\Pi}_{y,A}\, d\log\boldsymbol{A} + \boldsymbol{\Pi}_{y,H}\, d\log\boldsymbol{H},$$

where

$$\boldsymbol{\Pi}_{y,A} = \underbrace{\boldsymbol{\Psi}}_{\text{frictionless}} + \underbrace{\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\boldsymbol{I} - \underbrace{\mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)}_{\text{search cost}}\right)\boldsymbol{\Pi}_{\theta,A}}_{\text{tightness adjustment}},$$

$$\boldsymbol{\Pi}_{y,H} = \underbrace{\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}}_{\text{frictionless}} + \underbrace{\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right)\boldsymbol{\Pi}_{\theta,H}}_{\text{tightness adjustment}}.$$

The first-order impact of a shock on sectoral output can be split into two terms: one that captures the response in a frictionless economy and one that captures the effects of search and matching in the labor market through adjustments in tightness. If tightness remains unchanged after a shock, then the response of output in our model is identical to the response in a frictionless economy and depends only on production parameters. However, when tightness does change, our model's predictions deviate from the predictions of a frictionless model. How much output changes when tightness changes depends on search costs, $\mathcal{M}(\boldsymbol{I}+\mathfrak{T})$, which determine how much more of the workforce must be allocated to recruiting when tightness rises. In other words, $\boldsymbol{I} - \mathcal{M}(\boldsymbol{I}+\mathfrak{T})$ captures how much of any increase in employment leads to an increase in the productive workforce rather than to an increase in the number of recruiters.

Finally, our expression for changes in tightness allows us to derive the following expression for changes in unemployment across the $\mathcal{O}$ occupations.

PROPOSITION 3. *The expression for labor supply implies that first-order changes in end-of-period occupational unemployment are* $d\log \boldsymbol{U} = \left[d\log U_1, d\log U_2, \cdots, d\log U_{\mathcal{O}}\right]$ *follow*

$$d\log \boldsymbol{U} = \Pi_{U,A} d\log \boldsymbol{A} + \Pi_{U,H} d\log \boldsymbol{H},$$

where

$$\Pi_{U,A} = -\boldsymbol{U}^{-1}\boldsymbol{L}(\boldsymbol{I} - \mathcal{M})\,\Pi_{\theta,A},$$
$$\Pi_{U,H} = \boldsymbol{I} - \boldsymbol{U}^{-1}\boldsymbol{L}(\boldsymbol{I} - \mathcal{M})\,\Pi_{\theta,H},$$

and $\boldsymbol{U}$ *and* $\boldsymbol{L}$ *are* $\mathcal{O} \times \mathcal{O}$ *diagonal matrices with the number of unemployed and employed workers in the pre-shock equilibrium on the diagonal.*

Proposition 4 describes how relative prices respond to shocks to productivity, the labor force size, or factor supplies.

PROPOSITION 4. *The first-order responses of relative sectoral and factor prices are pinned down by labor force, technology, and factor supply shocks up to a numeraire. The first-order responses in sectoral prices satisfy*

$$\left(\boldsymbol{I} - \Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \mathcal{L}\right) d\log \boldsymbol{p} = \Pi_{p,A} d\log \boldsymbol{A} + \Pi_{p,H} d\log \boldsymbol{H},$$

where

$$\Pi_{p,A} = \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_A + \mathcal{M}\mathfrak{T}\Pi_{\theta,A}\right) - \boldsymbol{I}\right],$$
$$\Pi_{p,H} = \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_H + \mathcal{M}\mathfrak{T}\Pi_{\theta,H}\right)\right].$$

In our framework, relative prices are determined by market clearing in a perfectly competitive environment. The price of a good produced by a particular sector responds to price changes in all other sectors, with $\boldsymbol{\Psi}$ capturing the co-movement and interaction of prices throughout the production network. Moreover, prices are influenced by the effective labor costs. In models without search and matching, prices respond directly to change in wages, which corresponds to the $\boldsymbol{\Lambda}$ terms in the expressions above. However, in our model, wage adjustments impact prices through an additional tightness channel, which corresponds to the product of the matching elasticity matrix, the recruiter-producer matrix, and the $\boldsymbol{\Pi}_\theta$ matrices. This is attributed to how wage adjustments impact labor market tightness, which in turn affects the required number of recruiters to fill a given number of positions. In addition, the price is also directly linked to the productivity level in that sector. Thus, a productivity shock impacts the system of prices directly through production and indirectly through adjustments in tightness. Shocks to the size of the labor force impact prices solely through adjustments in labor market tightness.

At the disaggregate level, the interplay between labor market structure and production linkages significantly shapes price and allocations responses. In particular, changes in labor demand depend on how sectors connect to each other through the use of intermediate goods and what common occupations they hire from. These shifts in labor demand lead to changes in labor market tightness, which acts as an endogenous wedge that impacts both output and price responses.

3.2. The Aggregate Impact of Shocks

The expressions above tell us how output and unemployment across sectors and occupations change to first order. They are of independent interest because they describe which sectors and occupations are likely to be most affected by sectoral shocks given the labor market's production function parameters and features.

We can also use those expressions to describe how aggregate output and unemployment change in response to sector- or occupation-specific shocks. With Cobb-Douglas preferences, the first-order response of aggregate output is given by $d\log Y^{agg} = \boldsymbol{\sigma}' d\log \boldsymbol{y}$, where $\boldsymbol{\sigma}'$ is a $J \times 1$ vector of demand elasticities. Using Proposition 2, we arrive at the following result for first-order changes in aggregate output.

THEOREM 1. *Given idiosyncratic labor force shocks $d\log \boldsymbol{H}$ and productivity shocks $d\log \boldsymbol{A}$,* *the log change in real GDP is*

$$d \log Y^{agg} = \boldsymbol{\Pi}_{\boldsymbol{A},\boldsymbol{Y}^{agg}} d \log \boldsymbol{A} + \boldsymbol{\Pi}_{\boldsymbol{H},\boldsymbol{Y}^{agg}} d \log \boldsymbol{H},$$

where

$$\begin{aligned} \boldsymbol{\Pi}_{\boldsymbol{A},\boldsymbol{Y}^{agg}} &= \underbrace{\boldsymbol{\lambda}'}_{\text{frictionless}} + \underbrace{\boldsymbol{\lambda}' \boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \mathcal{M} \left(\boldsymbol{I} + \mathcal{T} \right) \right) \boldsymbol{\Pi}_{\theta,A}}_{\text{tightness adjustment}}, \\ \boldsymbol{\Pi}_{\boldsymbol{H},\boldsymbol{Y}^{agg}} &= \underbrace{\boldsymbol{\lambda}' \boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}}}_{\text{frictionless}} + \underbrace{\boldsymbol{\lambda}' \boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \mathcal{M} \left(\boldsymbol{I} + \mathcal{T} \right) \right) \boldsymbol{\Pi}_{\theta,H}}_{\text{tightness adjustment}}, \end{aligned}$$

and $\boldsymbol{\lambda} = \boldsymbol{\Psi}' \boldsymbol{\sigma}$ *denotes the sectors' sales shares.*

PROOF. The result follows from $d \log Y^{agg} = \boldsymbol{\sigma}' d \log \boldsymbol{y}$ and Proposition 2. □

Note that the matrix product between the Leontief inverse and the demand elasticities equals the sales shares. This property results from the household's maximization problem, the firms' profit maximization decision, goods market clearing, and the Cobb-Douglas production functions. Therefore, the aggregate impact of productivity and labor force shocks can be summarized as the sales-share-weighted impact of these shocks on sectoral output directly through production and indirectly through labor markets. This result is reminiscent of Hulten's theorem, which posits that in frictionless competitive economies, the first-order effect of a productivity shock to an industry on aggregate output equals that industry's sales share (Hulten 1978). In fact, we show in Section 3.3 how Hulten's theorem holds in a knife edge case of our model. Whenever tightness does not change in response to shocks, Hulten's theorem holds.

In addition to output, we are interested in how aggregate unemployment responds to sector-specific shocks. Using the results in Proposition 3, we can derive changes in aggregate unemployment.

COROLLARY 1. *Given idiosyncratic labor force shocks* $d \log \boldsymbol{H}$ *and productivity shocks* $d \log \boldsymbol{A}$, *the first-order response in aggregate unemployment is*

$$d \log U^{agg} = \boldsymbol{\Pi}_{\boldsymbol{A},\boldsymbol{U}^{agg}} d \log \boldsymbol{A} + \boldsymbol{\Pi}_{\boldsymbol{H},\boldsymbol{U}^{agg}} d \log \boldsymbol{H},$$

where

$$\Pi_{U,A} = -\mathbf{1}' \frac{\boldsymbol{U}}{U^{agg}} \boldsymbol{U}^{-1} \boldsymbol{L} \left(\boldsymbol{I} - \mathcal{M}\right) \Pi_{\theta,A},$$
$$\Pi_{U,H} = \mathbf{1}' \frac{\boldsymbol{U}}{U^{agg}} \left[\boldsymbol{I} - \boldsymbol{U}^{-1} \boldsymbol{L} \left(\boldsymbol{I} - \mathcal{M}\right) \Pi_{\theta,H}\right],$$

where $\mathbf{1}$ *is a* $\mathcal{O} \times 1$ *vector of ones and* U^{agg} *is the aggregate number of unemployed workers before the shock.*

PROOF. Follows from $d\log U^{agg} = \mathbf{1}' \frac{\boldsymbol{U}}{U^{agg}} d\log \boldsymbol{U}$ and Proposition 3. See Appendix A.2 for details. □

Intuitively, the change in aggregate unemployment is a weighted average of the first-order changes in sectoral unemployment.

3.3. The Role of Search and Matching in Amplifying the Response of Aggregate Output

Hulten (1978)'s theorem posits that in an efficient economy, the first-order effect of a productivity shock to an industry's aggregate output equals that industry's sales share. In this section, we compare our results to Hulten's theorem and examine how the interaction between production linkages and labor market search and matching impacts aggregate output. Specifically, we analyze the conditions under which Hulten's theorem holds as well as when the labor market structure interacts with production linkages to amplify the aggregate impact of shocks.

In the discussion following Proposition 2, we described how labor market inefficiencies impact output through adjustments in tightness. Tightness generates an additional wedge between wages and the marginal product of labor. This additional wedge exists because in tighter labor markets, firms must dedicate more resources to recruiting, which increases the marginal cost of labor. When network-price-adjusted wages change in exact proportion to the marginal product of labor, tightness remains constant. Because tightness is not affected, search costs do not vary, thereby upholding Hulten's theorem.

COROLLARY 2. *Hulten's theorem holds for technology shocks whenever network-price-adjusted wages change in exact proportion to the network-adjusted marginal product of labor, that is, when*

$$d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = \mathcal{L} d\log MP,$$

where $d\log MP$ is a matrix of changes to the marginal product of each type of labor in each sector. Furthermore, when this condition holds, aggregate changes in response to labor force and factor supply shocks do not depend on the costly search-and-matching process.

PROOF. The derivative of production with respect to labor inputs, along with the firms' first-order conditions, imply that the network-adjusted marginal product of labor satisfies

$$\mathcal{L} d\log \mathrm{MP} = \mathfrak{T}\mathcal{M} d\log \theta + d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}.$$

Imposing that network-price-adjusted wage changes are exactly proportional to changes in the network-adjusted marginal product of labor implies

$$\mathfrak{T}\mathcal{M} d\log \theta = 0.$$

$\mathfrak{T}\mathcal{M}$ is a diagonal matrix with non-zero diagonal elements. Therefore,

$$d\log \theta = 0.$$

Since search-and-matching operates through changes in $d\log \theta$, this implies that search-and-matching has no impact on the propagation of shocks. In particular, in this case

$$d\log Y = \lambda' \left[d\log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \boldsymbol{H} \right].$$

The aggregate output response to technology shocks is $\lambda' d\log \boldsymbol{A}$, which is exactly Hulten's theorem. The aggregate output response to the other shocks depends only on production parameters; the labor market structure plays no role. □

When wages do not respond to shut off changes in tightness, search and matching impacts the aggregation of idiosyncratic shocks. The following corollary formally characterizes the search channel of idiosyncratic shocks.

COROLLARY 3. *When wages do not respond exactly proportionally to the network-adjusted marginal product of labor, a matching labor market structure generates deviations from Hulten's theorem, captured by*

$$\Pi_{search,A} = \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathfrak{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi - \Lambda_A\right),$$
$$\Pi_{search,H} = \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathfrak{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\left[\mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] - \Lambda_H\right).$$

PROOF. Follows from Theorem 1 and 2. □

The search channel is a product of sales shares λ, the labor elasticity matrix $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}$, the search cost term $(\boldsymbol{I} - \mathcal{M}(\boldsymbol{I} + \mathcal{T}))$, and the first-order response in labor market tightness. We want to examine whether the search channel amplifies the aggregate impact of shocks. Before we proceed, we first define amplification.

DEFINITION 1. *The search channel* ***amplifies*** *the shock's impact if* $\Pi_{search,x} > 0$ *element-wise.*

When this definition holds, a shock to any sector or occupation also impacts output through the search channel. This impact, as the name amplification suggests, has the same sign as the shock.

In theory, whether amplification occurs depends on the magnitudes of the matching elasticities $\mathcal{M}$, the recruiter-producer ratio $\mathcal{T}$, production structures, occupational structure, and the wage schedules. We believe that the empirically plausible case is that wages are somewhat rigid. By this, we mean the following.

DEFINITION 2. *Wages are* ***rigid*** *in response to*

- *technology shocks if* $\mathcal{L}\Psi - \Lambda_A$ *is non-negative, with one strictly positive element in each column, and*
- *labor force shocks if* $\mathcal{L}\Psi\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} - \boldsymbol{I} - \Lambda_H$ *is non-negative, with one strictly positive element in each column.*

In words, wages are rigid when they adjust less than the occupation and network-adjusted labor productivity. This definition of rigidity is less restrictive than those typically found in the search-and-matching literature. Take the wage response to productivity as an example. In Hall (2005), Blanchard and Galí (2010), and Michaillat (2012), wages are rigid if they adjust less than proportionally to changes in productivity. However, in a production network—take $\mathcal{L} = \boldsymbol{I}$, for example—the diagonal elements of the Leontief inverse Ψ can often be greater than 1. Therefore, a wage schedule that is viewed as flexible in the conventional sense can be seemed as rigid here. We also consider wages as rigid overall even if all rows in $\mathcal{L}\Psi - \Lambda_{\boldsymbol{A}}$ are zero but one. That is, our definition of wage rigidity allows wages to adjust exactly proportionally to productivity, i.e., to be flexible, in every sector but one. Existing estimates suggest that wages are indeed rigid by our definition. For instance, Haefke et al. (2013) estimate the wage elasticity to productivity to be around 0.8 for new hires and 0.24 for all workers.[9]

Now, we characterize when the search channel amplifies different types of shocks:

PROPOSITION 5. *If* $(\boldsymbol{I}+\mathcal{T})^{-1} > \mathcal{M}$ *and wages are rigid, the search channel amplifies the shock.*

PROOF. When $(\boldsymbol{I}+\mathcal{T})^{-1} > \mathcal{M}$, $(\boldsymbol{I}-\mathcal{M}(\boldsymbol{I}+\mathcal{T}))$ is greater than 0 on the diagonals, the equilibrium adjustment coefficients $\left[\boldsymbol{I}-\mathcal{M}-\Xi_\theta\right]^{-1}$ are non-negative element-wise and positive on the diagonals (shown in Appendix A.3). □

When wages are rigid and labor markets are slack, the search channel amplifies the aggregate impact of idiosyncratic shocks on output. Intuitively, rigid wages make workers more attractive to firms following a positive technology shock, as wages adjust less than the increase in labor productivity. In slack labor markets, fewer recruiters are needed to hire additional workers, leading to an increase in employment and a more substantial output response than predicted under Hulten's theorem. Conversely, in tight labor markets where recruiting costs are high, the additional recruiters needed to hire an additional worker may outweigh the benefits. In such scenarios, the search channel could dampen rather than amplify the effect shocks. For instance, firms attempting to take advantage of the beneficial wages by posting additional vacancies may end up hiring fewer productive workers and more recruiters. In this case, employment increases but output actually increases less than what Hulten's theorem would suggest.

Quantitatively interpreting the condition $(\boldsymbol{I}+\mathcal{T})^{-1} > \mathcal{M}$ is difficult given the static setup of our model. To ground our analysis on the effect of labor market tightness on shock propagation based on empirically plausible model parameters, we extend the model by modifying the labor market block with a steady-state, balanced flows assumption in the Online Appendix. We allow some workers to be employed initially but assume the economy starts out in a steady state where the number of workers who flow into unemployment equals those who flow out. We then analyze how steady-state employment changes after a shock. All key equations remain qualitatively identical to our static formulation, except that the model parameters can now be interpreted in the context of empirical estimates.

The equivalent condition in our balanced flows specification is $\boldsymbol{u}(\boldsymbol{u}+\mathcal{T})^{-1} > \mathcal{M}$, where $\boldsymbol{u}$ is a matrix with the unemployment rates in each occupation along the diagonal. Instead of requiring $\boldsymbol{I}-\mathcal{M}(\boldsymbol{I}+\mathcal{T}) > 0$ on the diagonals, we now need $\boldsymbol{u}-\mathcal{M}(\boldsymbol{u}+\mathcal{T}) > 0$

[9] One important case where our definition of wage rigidity does not apply is for revenue sharing, as in Diamond (1982a). This type of revenue sharing results in wage changes that are proportional to productivity changes. Nevertheless, our definition of rigidity can still hold even if wages are determined through revenue sharing in all sectors except one.

because the initial unemployment rate is no longer 1. Thus, the dependence of amplification on labor market tightness is clearer. Suppose the matching elasticity is 0.7 and the recruiter-producer rate equals 2.3%, both in the range of plausible estimates from Petrongolo and Pissarides (2001) and Landais et al. (2018). If the unemployment rate is 6%, then the search channel amplifies productivity shocks. However, if it is 3%, the search channel can dampen the productivity shock.[10]

4. Calibration

So far, we have established qualitatively that incorporating search frictions in a production network economy can amplify or dampen the disaggregate and aggregate economic impact of microeconomic shocks, depending on labor market tightness. In this section, we test the quantitative importance of the search channel by calibrating our model to the U.S. economy and exploring the response of output and unemployment to productivity shocks.

To allow a more direct fit to the data, we make two minor adjustments to the model outlined in Section 2. First, we account for two additional factors of production— capital and energy—allowing us to match the production elasticities to intermediate inputs and labor. Second, rather than assuming that all workers begin unemployed, we assume the labor market is in a steady state with balanced flows from employment to unemployment and back. This allows for a more direct match between the data on employment and unemployment by sector and the model, without substantially changing the structure of the model presented above. We outline the specific adjustments required to incorporate balanced labor market flows in the Online Appendix.

4.1. Labor Market Parameters

Given the large number of parameters resulting from the interaction between the production network and labor markets, we relegate the bulk of our calibration procedure to the Online Appendix. This and the following section briefly discuss data sources and calibration for the key labor market and production parameters.

In this calibration exercise, we split the labor force into major occupation categories and allow firms in each sector to use a mix of the different major occupations in production. Workers are constrained to remain in one occupation but are not constrained

[10] In the Online Appendix, we show that when there is just one occupation per sector, $\boldsymbol{u} - \mathcal{M}(\boldsymbol{u} + \mathcal{T}) < 0$, and wages are rigid and proportional to $\boldsymbol{L\Psi}$, the search channel dampens the shock.

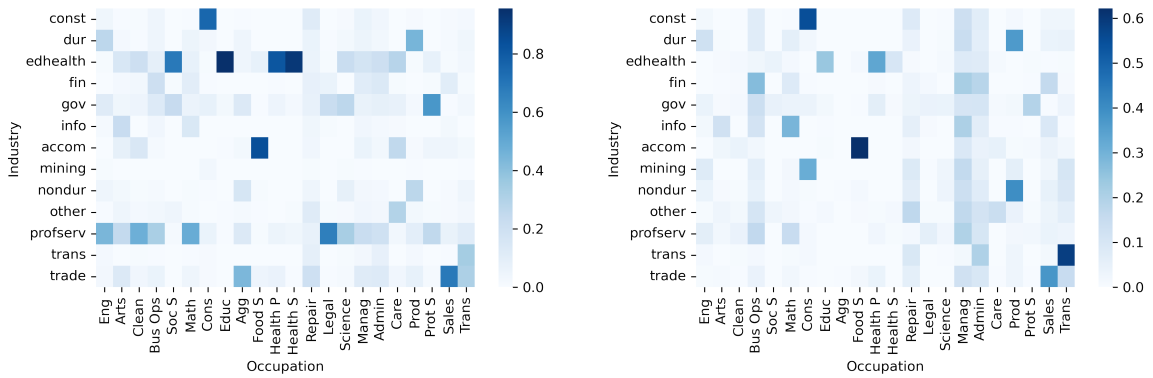
to remain in just one sector due to firms in different sectors using multiple occupations as an input. In general, our model is flexible, allowing for modifications to the labor market specification to adapt to various scenarios. For example, labor markets can be assumed to be fully rigid geographically to study the localized impact of industry shocks. The quantitative importance of our search channel remains robust across different labor market specifications. For instance, our findings still hold if each sector hires just one unique occupation, eliminating sector-to-sector worker flows.

We choose an occupation level calibration because sector-to-sector labor transitions are a potentially important feature of the labor market in many countries. For instance, in the U.S. about 12%–20% of jobs switchers also change industries at the one-digit level (Kambourov and Manovskii 2008; Parrado et al. 2007). Neffke et al. (2017) find that nearly 59% of German job movers change industry at the most aggregate German industry grouping. These sector-to-sector flows are possible in an occupation calibration because even though workers remain in the same occupation, they may transition to another sector when they switch jobs.

Allowing for realistic sector-to-sector flows of workers comes the cost of high data requirements at the major occupational level. In this calibration, we address these requirements by imputing certain occupational parameters from sector-level data. In particular, we use vacancy and hiring data from the JOLTS, which provides survey-based measures of job openings and hires at a monthly frequency, available from December 2000 to February 2023. These survey data are available for 13 industries that roughly correspond to the two-digit NAICS industry classifications. We use sector-level unemployment data from the CPS. These data cover 13 sectors at a monthly frequency over the same time frame as the JOLTS data. Finally, we use occupation-level data from the BLS's 2021 OEWS for sectors at the two- and three-digit NAICS classification level to construct sector-by-sector major occupational employment and wages.

We define a major occupation to be an occupation at the two-digit Standard Occupational Classification level and impute occupation level parameters where they are not available using sector level data and occupation-by-sector employment shares. We assume that the total number of unemployment, vacancy, and recruiters for an occupation is the sum of unemployment, vacancy, and recruiters across sectors, weighted by the sectors' labor expenditure shares of that particular occupation. Figure 1 plots the resulting $\mathcal{L}$ and $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ matrices, which respectively capture each sectors' importance as an employer of each occupation and each occupations importance in the production of each sector.

FIGURE 1. Occupation shares and occupation elasticity for different industries.



A. Occupation share

B. Occupation elasticity of production

Notes: Panel A shows the share of given occupations used by different industries. The $(i, j)^{\text{th}}$ element denotes the share of occupation j employed by sector i. Each column sums to 1. Panel B presents the occupation elasticity of production. The $(i, j)^{\text{th}}$ element denotes the share of labor expenditure that sector i spends on workers in occupation j. Each row sums to 1.

We use the number of HR workers employed in each sector as a proxy for recruiting efforts, which underestimates the total recruiting activities, as non-HR workers may also participate in hiring new workers. This underestimation of recruiting efforts could weaken the power of the search-and-matching channel in our model, and we therefore view it as a conservative assumption.

Finally, we estimate the occupation-specific matching elasticities from imputed occupation hires, vacancies, and unemployment at the monthly frequency. The matching efficiency, ϕ_o, does not affect any of our results to first order. We therefore allow the matching efficiency to equal to the residual from this estimation, which ensures that the number of hires does not exceed the number of vacancy postings or the number of unemployed workers.

4.2. Production parameters

Thanks to our Cobb-Douglas specification, our model's production parameters are easy to map into readily available data on sector level input usage. The BEA Make and Use tables at the three-digit NAICS level allow us to calculate the intermediate input intensity of each sector, the labor intensity of each sector, and the elasticity of final consumption demand to each sector's output. We use employee compensation, recorded in the Use table, to calculate labor elasticities.

In our calibration, we consider two additional factors of production: capital and energy. Capital and energy shares for the two- and three-digit NAICS classification are available in the BEA-BLS Integrated Industry-Level Production Accounts (KLEMS). We can extend our model to include additional non-labor factors of production without meaningfully changing any of the structure or results in the previous sections. We include additional factors of production in our calibration to obtain a better match to the labor and intermediate input elasticities, which play a key role in the effects of sector-specific shocks.

Although the JOLTS and the CPS include labor market data at a level corresponding roughly to the two-digit NAICS classification, this correspondence is not always exact. Whenever this is the case, we use data at the two- or three-digit NAICS classification level and aggregate back up to match the 13 CPS industries (Horowitz and Planting 2009).

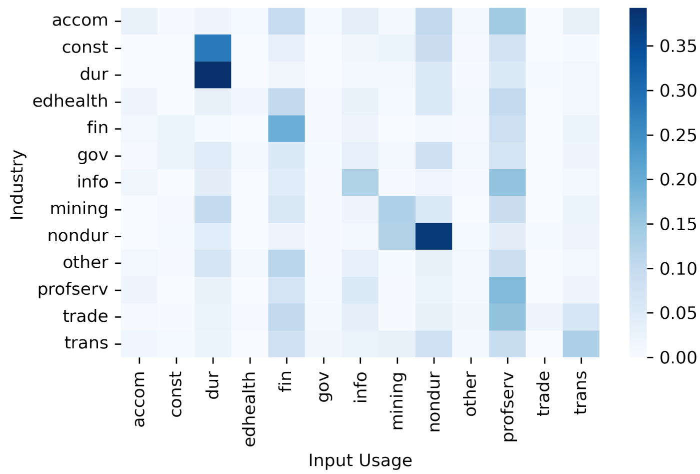
4.3. Wage Schedule

In our model economy, wages play an important role in how shocks propagate, and how they adjust determines the response in tightness after an economic shock. In fact, as Theorem 2 shows, for the right assumption about wages, search frictions can have no effect whatsoever on shock propagation. However, in general, the degree of wage flexibility required by Theorem 2 is rare. Haefke et al. (2013) estimate the elasticity of wages of job movers with respect to productivity to be 0.7. We view this as a reasonable benchmark for the degree of wage flexibility but show below that our results are robust to alternative assumptions about wage flexibility. In the context of our model, we assume that the change in real wage is 0.7 times the log change in marginal product of labor:

$$d\log \boldsymbol{w} - \mathcal{L} d\log\ \boldsymbol{p} = 0.7 d\log \boldsymbol{MP}.$$

Figure 2 reports the calibrated input-output matrix $\boldsymbol{\Omega}$, showing that a sector's production usually relies on intermediate goods produced by other sectors. These parameters are calibrated by computing each sector's expenditure share on commodities produced by other sectors.

FIGURE 2. Input-output table for major sectors, roughly corresponding to the NAICS two-digit sectors.



Notes: The $(i, j)^{\text{th}}$ element on the heat map denotes the share of sector i's revenue spent on intermediate goods produced by sector j.

We show in the Online Appendix that our results are robust to alternative wage assumptions.

4.4. The Effect of Productivity Shocks

In this section we characterize the quantitative importance of combining search-and-matching with production networks by introducing productivity shocks. To assess the quantitative importance, we need appropriate baseline economies to compare our results to. To this end, we report results for three specifications in our figures below:

(i) Linkages and Frictions: Our full model featuring production linkages and search-and-matching.

(ii) Linkages Only: A network model without search-and-matching ($\mathcal{M} = \boldsymbol{I}$).

(iii) Frictions Only: A multi-sector search-and-matching model with no production linkages ($\boldsymbol{\Psi} = \boldsymbol{I}$).

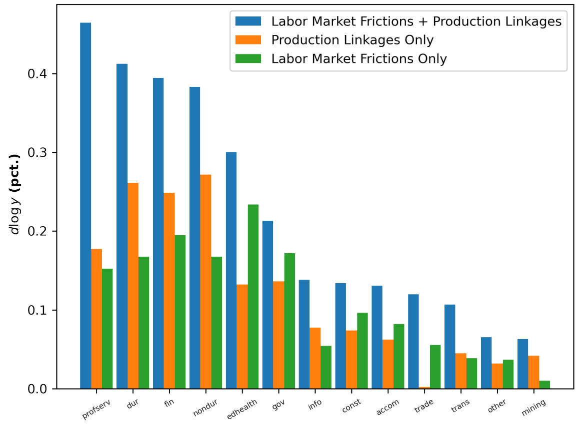
The top panel of Figure 3 plots the response of aggregate output and unemployment to a 1% productivity shock in each of our 13 sectors. The bottom panel plots the response of output and unemployment across sectors and occupations to a 1% productivity shock in the durable manufacturing sector. The orange bars represent the benchmark with only production linkages, while the green bars represent the benchmark with only labor market frictions. The blue bars represent our model, which features both production linkages and search-and-matching frictions.

Both output and unemployment respond more to technology shocks once we account for both production linkages and labor market frictions. This is true across all sectors, and the amplification is substantial. For instance, in our model, aggregate output and unemployment respond more than twice as much to a productivity shock in the professional and business services sector than in either of the two benchmark cases.

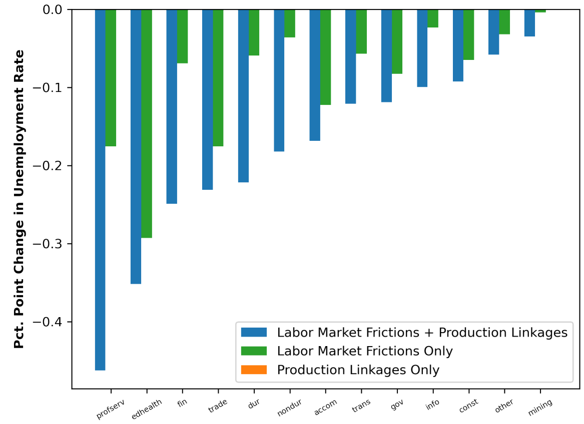
More interestingly, accounting for both production linkages and labor market frictions changes the relative importance of different sectors for aggregate output and unemployment. A researcher focusing on a multisector search model with occupational labor markets and no production linkage might conclude that the education and health services sector is the most important for both aggregate output and unemployment. However, because it is not an important supplier to other sectors in the production economy, the education and health services sector is only the fifth most important sector for aggregate output and the second most important sector for aggregate unemployment, when we account of production linkages.

Conversely, a researcher focusing on an efficient production network would conclude that the non-durable manufacturing sector is the most important for output. However, because the non-durable manufacturing sector primarily employs production workers, who are not a particularly important labor input for any non-manufacturing sector, once we account for occupational labor market frictions it is only the fourth most important sector for output. On the other hand, they would overlook the importance of the wholesale trade sector because of its relatively small Domar weight—its output is not not a major component of final consumption. Once we account for the occupational labor market structure, because it is an important employer of sales and transportation workers, this sector has non-negligible effects on aggregate unemployment, and therefore also on aggregate output. In other words, network centrality is no longer a sufficient measure of a sectors importance to aggregate output once we allow for a more complex labor market structure.

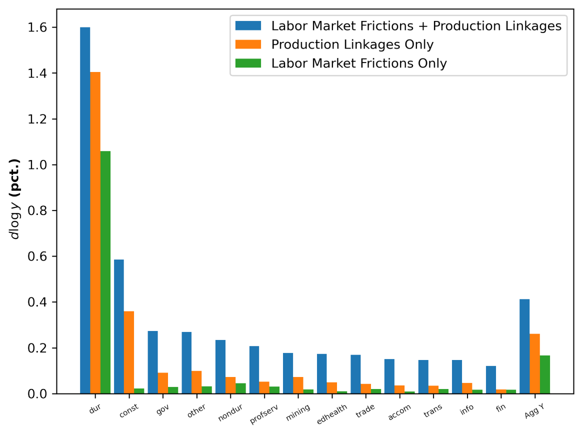
FIGURE 3. Response of output and the unemployment rate to a 1% shock to technology in the durable manufacturing sector.



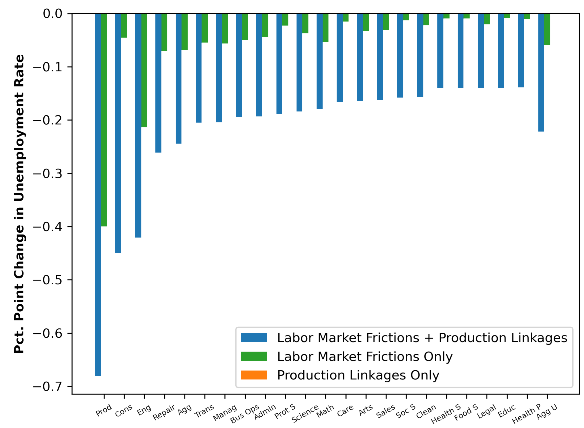
A. Aggregate output response



B. Aggregate unemployment rate response



C. Sectoral output response - durables shock

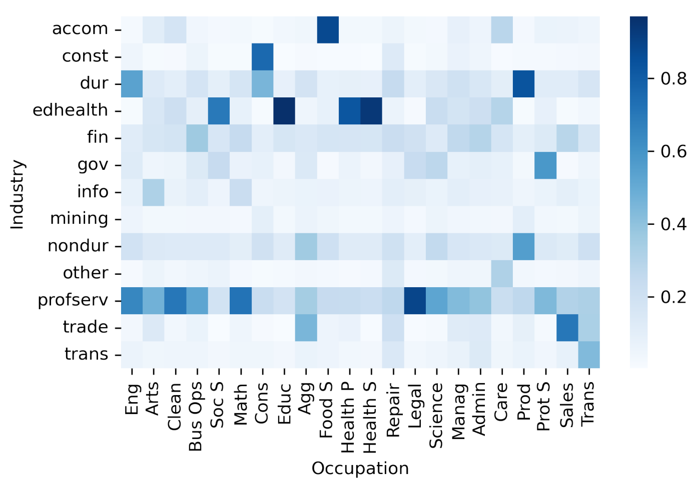


D. Unemployment rate response - durables shock

Combining production linkages with a realistic segmented and frictional labor market setup therefore improves our understanding of which sectors are most important for aggregate output and unemployment. We account for both each sectors importance as a direct and indirect supplier to other sectors and labor market linkages between sectors that substantially alter how shocks propagate and aggregate. Crucially, the effects of productivity shocks once we combine network linkages with search-and-matching is not simply the sum of the effects in the two baseline models. Consider, for instance, a shock to the professional and business services sector. Our model predicts that the effect of this shock on aggregate output is larger than the sum of the effects in our two baselines.

Figure 4 plots the transpose of $\mathcal{L}\Psi$ can help explain why the professional services sector is so important in our model. $\mathcal{L}\boldsymbol{\Psi}$ captures the network adjusted productivity gain after a shock $d\log \boldsymbol{A}$, reflecting both how important the shocked sector is to each other sector in the production network, $\boldsymbol{\Psi}$, and how important each sector is to each type of labor. Despite being only the fourth most important sector for output in a pure production network and the third most important sector for unemployment in a multi-sector search-and-matching model, the professional services sector is particularly important because it has a large impact on the network adjusted productivity of many occupations. It is itself a large employer of many occupations, and is connected through the production network to other large employers, boosting the effective productivity of many types of labor. While a multi-sector search-and-matching model captures the first round employment effects, it fails to capture the second round effects coming from the sectors that the professional services sector is connected to. The production network model accounts for linkages through intermediate inputs, but fails to account for the employment effects.

FIGURE 4. Occupation-share-adjusted Leontief inverse.



Notes: The $(i, j)^{\text{th}}$ element on the heat map denotes the network-adjusted productivity gain for workers in occupation j in response to a productivity shock in sector i.

The bottom panel of figure 3, which plots the response of sector level output and occupation level unemployment to a 1% productivity shock in durable manufacturing—the closest analog to our example from the introduction—demonstrates that the interesting dynamics persist at the dis-aggregate level, particularly for unemployment. Allowing for both production linkages and labor market frictions substantially amplifies the response of unemployment in occupations important to durable manufacturing and leads to widespread declines in unemployment across all occupations. In our full model, the unemployment rate declines by 0.4 percentage points for engineers, by 0.7 percentage points for production workers, and by at least 0.1 percentage points in every occupation. As a result, the aggregate unemployment rate falls by 0.2 percentage points.

5. Conclusion

Modern economies are characterized by intricate production networks and labor markets marked by frictional and segmentation. Our analysis demonstrates that considering both aspects shifts our understanding of shock transmission, both quantitatively and qualitatively. The extent to which matching frictions impact network propagation depends on wage assumptions, revealing that, generally, labor market frictions amplify productivity shocks except in scenarios of extreme labor market tightness. By calibrating our model to U.S. data, we find that this effect is be sizable and changes the relative importance of different sectors for aggregate output and unemployment.

This framework opens up several venues for future research. First, it would be interesting to explore the business cycle implications of our model, such as how the interaction between production linkages and search frictions impact the cyclical movements of workers across sectors, occupations, and regions. Additionally, exploring the non-linearities and higher-order propagation of our model warrants further investigation. Last, while our current model cannot speak to the effect on the price level, we hope to extend the model by incorporating nominal rigidities to help paint a realistic picture of the post-COVID inflation episode, which involves interesting interactions between production sectors and labor markets. We are currently working on an extension to our model to address the post-Covid inflation episode.

References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, "The Network Origins of Aggregate Fluctuations," *Econometrica*, 2012, *80* (5), 1977–2016.

Baqaee, David and Emmanuel Farhi, "Supply and Demand in Disaggregated Keynesian Economies with an Application to the COVID-19 Crisis," *American Economic Review*, May 2022, *112* (5), 1397–1436.

Baqaee, David Rezza and Emmanuel Farhi, "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 2020, *135* (1), 105–163.

Benigno, Pierpaolo and Gauti Eggertsson, "It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve," 2023.

Bertheau, Antoine, Edoardo Maria Acabbi, Cristina Barceló, Andreas Gulyas, Stefano Lombardi, and Raffaele Saggio, "The Unequal Consequences of Job Loss across Countries," *American Economic Review: Insights*, 2023, *5* (3), 393–408.

Bewley, Truman, "Fairness, Reciprocity, and Wage Rigidity," in Herbert Gintis, Samuel Bowles, Robert Boyd, and Ernst Fehr, eds., *Moral Sentiments and Material Interests: The Foundations of Cooperation in Economic Life*, The MIT Press, June 2005, p. 0.

Bigio, Saki and Jennifer La'O, "Distortions in Production Networks," *The Quarterly Journal of Economics*, 2020, *135* (4), 2187–2253.

Blanchard, Olivier and Jordi Galí, "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, April 2010, *2* (2), 1–30.

Borgschulte, Mark and Paco Martorell, "Paying to Avoid Recession: Using Reenlistment to Estimate the Cost of Unemployment," *American Economic Journal: Applied Economics*, July 2018, *10* (3), 101–127.

di Giovanni, Julian, Sebnem Kalemli-Ozcan, Alvaro Silva, and Muhammed Ali Yildirim, "Pandemic-Era Inflation Drivers and Global Spillovers," 2023.

Diamond, Peter A, "Aggregate Demand Management in Search Equilibrium," *JOURNAL OF POLITICAL ECONOMY*, 1982.

Diamond, Peter A., "Wage Determination and Efficiency in Search Equilibrium," *The Review of Economic Studies*, 1982, *49* (2), 217–227. Publisher: [Oxford University Press, Review of Economic Studies, Ltd.].

Fernandez-Villaverde, Jesus, "Solving High-Dimensional Dynamic Programming Problems using Deep Learning," 2020.

Haefke, Christian, Marcus Sonntag, and Thijs van Rens, "Wage rigidity and job creation," *Journal of Monetary Economics*, November 2013, *60* (8), 887–899.

Hall, Robert E., "Employment Fluctuations with Equilibrium Wage Stickiness," *The American Economic Review*, 2005, *95* (1), 50–65.

Horowitz, Karen J. and Mark A. Planting, "Concepts and Methods of the U.S. Input-Output Accounts," *Bureau of Economic Analysis*, 2009.

Hulten, C. R., "Growth Accounting with Intermediate Inputs," *The Review of Economic Studies*, 1978, *45* (3), 511–518.

Humlum, Anders, "Robot Adoption and Labor Market Dynamics," 2021, p. 69.

Jacobson, Louis S., Robert J. LaLonde, and Daniel G. Sullivan, "Earnings Losses of Displaced Workers," *The American Economic Review*, 1993, *83* (4), 685–709. Publisher: American Economic Association.

Jones, Charles I., "Misallocation, Economic Growth, and Input-Output Economics," 2011.

Kambourov, Gueorgui and Iourii Manovskii, "Rising Occupational and Industry Mobility in the United States: 1968–97," *International Economic Review*, 2008, *49* (1), 41–79.

Landais, Camille, Pascal Michaillat, and Emmanuel Saez, "A Macroeconomic Approach to Optimal Unemployment Insurance: Theory," *American Economic Journal: Economic Policy*, May 2018, *10* (2), 152–181.

La'O, Jennifer and Alireza Tahbaz-Salehi, "Optimal Monetary Policy in Production Networks," *Econometrica*, 2022, *90* (3), 1295–1336.

Liu, Ernest, "Industrial Policies in Production Networks," *The Quarterly Journal of Economics*, 2019, *134* (4), 1883–1948.

Long, John B. and Charles I. Plosser, "Real Business Cycles," *Journal of Political Economy*, 1983, *91* (1), 39–69.

Mas, Alexandre and Amanda Pallais, "Labor Supply and the Value of Non-Work Time: Experimental Estimates from the Field," *American Economic Review: Insights*, June 2019, *1* (1), 111–126.

Michaillat, Pascal, "Do Matching Frictions Explain Unemployment? Not in Bad Times," *The American Economic Review*, June 2012, *102* (4), 1721–1750. Num Pages: 30 Place: Nashville, United States Publisher: American Economic Association Section: Shorter Papers.

— **and Emmanuel Saez**, "Aggregate Demand, Idle Time, and Unemployment," *Quarterly Journal of Economics*, May 2015, *130* (2).

— **and** — , "Beveridgean unemployment gap," *Journal of Public Economics Plus*, 2021, *2*, 100009.

Mortensen, Dale T., "The Matching Process as a Noncooperative Bargaining Game," in "The Economics of Information and Uncertainty," University of Chicago Press, 1982, pp. 233–258.

—, "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review*, December 1982, *72* (5), 968. Publisher: American Economic Association.

Neffke, Frank M.H., Anne Otto, and Antje Weyh, "Inter-industry labor flows," *Journal of Economic Behavior & Organization*, 2017, *142*, 275–292.

Parrado, Eric, Asena Caner, and Edward N. Wolff, "Occupational and industrial mobility in the United States," *Labour Economics*, 2007, *14* (3), 435–455.

Petrongolo, Barbara and Christopher A Pissarides, "Looking into the Black Box: A Survey of the Matching Function," 2001.

Pissarides, Christopher A., "Search Intensity, Job Advertising, and Efficiency," *Journal of Labor Economics*, January 1984, *2* (1), 128–143.

—, "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *American Economic Review*, September 1985, *75* (4), 676–690. Publisher: American Economic Association.

Pissarides, Christopher A, *Equilibrium Unemployment Theory*, 2nd ed., The MIT Press, February 2017.

Rubbo, Elisa, "Networks, Phillips Curves, and Monetary Policy," *Econometrica*, 2023, *91* (4), 1417–1455. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA18654.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *The American Economic Review*, 2005, *95* (1), 25–49.

Appendix A. Proofs

A.1. Proofs for Propositions 1, 2, 3, and 4

A.1.1. Tightness Propagation

Labor market clearing implies that changes in labor demand have to equal changes in labor supply:

$$d\log L_o^s(\theta, \boldsymbol{H}) = d\log L_o^d(\theta, \boldsymbol{A}).$$

$$\varepsilon_{\theta_o}^{\mathcal{F}_o} d\log\theta_o + d\log H_o = \sum_{i=1}^{J} \frac{l_{io}}{L_o^d} d\log l_{io}(\theta_o)$$

For every sector i we have

$$d\log l_{io}(\theta_o) = d\log p_i - d\log w_o + d\log y_i$$

Which, stacking over occupations, implies that

$$d\log \boldsymbol{L^d}(\theta) = \mathcal{L} d\log \boldsymbol{y} - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right]$$

since $\sum_{i=1}^{J} \frac{l_{io}}{L_o^d} = 1$ for all o. Plugging in for $d\log \boldsymbol{y}$ gives

$$\begin{aligned} d\log \boldsymbol{L^d}(\theta) = \mathcal{L}\Psi \left[d\log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\mathcal{F} + \mathfrak{Q}\mathfrak{T})\, d\log\theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \boldsymbol{H}\right] \\ - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right] \end{aligned}$$

Labor market clearing implies

$$\begin{aligned} \mathcal{F} d\log\theta + d\log \boldsymbol{H} = \mathcal{L}\Psi \left[d\log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\mathcal{F} + \mathfrak{Q}\mathfrak{T})\, d\log\theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \boldsymbol{H}\right] \\ - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right] \end{aligned}$$

Which pins down first order changes in log tightness as

$$d\log\theta = \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\mathcal{L}\Psi d\log \boldsymbol{A} - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right] + \left[\mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] d\log \boldsymbol{H}\right]$$

Where $\Xi_\theta = \mathcal{L}\Psi\varepsilon^{f}_{\boldsymbol{N}}[\mathcal{F} + \mathfrak{Q}\mathfrak{T}]$. Rearranging yields Proposition 1.

A.1.2. Output Propagation (Proposition 2)

The following relationship between Domar weights, $\lambda_i = \frac{p_i y_i}{G}$, and sales shares must hold in every sector:

$$p_j x_{ij} = \varepsilon^{f_i}_{x_{ij}} \lambda_i G.$$

Log-linearizing and using that this condition must hold in any two sectors i and j we can write

$$d\log x_{ij} = d\log\lambda_i - d\log\lambda_j + d\log y_j$$

Plugging back into the production function,

$$d\log y_i = d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} d\log y_j$$

Using the definition of labor demand,

$$\sum_i \frac{l_{io}}{L_o} d\log N_{io} = d\log L^d_o + \tau_o(\theta_o)\varepsilon^{\mathcal{Q}_o}_{\theta_o} d\log\theta_o$$

Log linearizing the labor usage ratio for an occupation by two different sectors

$$\frac{l_{io}}{l_{jo}} = \frac{\varepsilon^{f}_{N_{io}}\lambda_i}{\varepsilon^{f}_{N_{jo}}\lambda_j}$$

gives

$$d\log l_{io} = d\log l_{jo}$$

Also since, $d\log l_{io} = d\log N_{io} + d\log(1+\tau_o(\theta_o)) = d\log l_{jo} = d\log N_{jo} + d\log(1+\tau_o(\theta_o))$, we have that $d\log N_{io} = d\log N_{jo}$.

Using the labor market clearing condition, and the definition of labor supply,

$$d\log N_{io} \underbrace{\sum_k \frac{l_{ko}}{L_o}}_{=1} = \left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o)\varepsilon_{\theta_o}^{\mathcal{Q}_o}\right) d\log\theta_o + d\log H_o$$

Plugging this back into the linearized production function gives:

$$\begin{aligned} d\log y_i = d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon_{N_{io}}^{f_i} \left[\left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o)\varepsilon_{\theta_o}^{\mathcal{Q}_o}\right) d\log\theta_o + d\log H_o\right] \\ + \sum_{j=1}^{J} \varepsilon_{x_{ij}}^{f_i} d\log y_j \end{aligned}$$

Stacking over sectors gives,

$$d\log \boldsymbol{y} = d\log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\mathfrak{F} + \mathfrak{Q}\mathfrak{T})\, d\log\theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \boldsymbol{H} + \boldsymbol{\Omega} d\log \boldsymbol{y}$$

Which implies

$$d\log \boldsymbol{y} = \boldsymbol{\Psi}\left(d\log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\mathfrak{F} + \mathfrak{Q}\mathfrak{T})\, d\log\theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \boldsymbol{H}\right)$$

Rearranging and combining with Proposition 1 yields Proposition 2.

A.1.3. Unemployment Propagation

Occupational unemployment is given by:

$$U_o = H_o - L_o,$$

which implies that:

$$d\log U_o = \frac{H_o}{U_o} d\log H_o - \frac{L_o}{U_o} d\log L_o.$$

Using the definition for labor supply, we have that:

$$\begin{aligned} d\log U_o &= \frac{H_o}{U_o} d\log H_o - \frac{L_o}{U_o}\left((1-\eta_o)\, d\log\theta_o + d\log H_o\right) \\ &= d\log H_o - \frac{L_o}{U_o}(1-\eta_o)\, d\log\theta_o \end{aligned}$$

Alternatively, we can rewrite the expression above in terms of job-finding rates:

$$d \log U_o = d \log H_o - \frac{f_o(\theta_o)}{1 - f_o(\theta_o)} (1 - \eta_o)\, d \log \theta_o.$$

Rearranging and stacking over occupations yields Proposition 3.

A.1.4. Price Propagation (Proposition 4)

Plugging in Equation 4 and Equation 3, the first order conditions for optimal input usage, into the log-linearized production function, using the fact that the sum of elasticities equals one for constant returns to scale technolog, y and $\varepsilon^{f_i}_{x_{ij}} d \log \varepsilon^{f_i}_{x_{ij}} = d\varepsilon^{f_i}_{x_{ij}}$, gives

$$d \log y_i = \left[d \log y_i + d \log p_i\right] \underbrace{\left[\sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}}\right]}_{=1 \text{ by crts}} + \underbrace{\left[\sum_{o=1}^{\mathcal{O}} d\varepsilon^{f_i}_{N_{io}} + \sum_{j=1}^{N} d\varepsilon^{f_i}_{x_{ij}}\right]}_{=0 \text{ by crts}}$$
$$- \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d \log w_o + d \log(1 + \tau_o(\theta_o))\right] - \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d \log p_j\right] + d \log A_i,$$

Rearranging terms gives

$$d \log p_i = \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d \log w_o - \tau_o(\theta_o) \varepsilon^{\mathcal{Q}_o}_{\theta_o} d \log \theta_o\right] + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} \left[d \log p_j\right] - d \log A_i$$

Stacking equations over sectors, we can write

$$d \log \boldsymbol{p} = \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d \log \boldsymbol{w} - \mathfrak{Q}\mathfrak{T} d \log \theta\right] + \boldsymbol{\Omega} d \log \boldsymbol{p} - d \log \boldsymbol{A}$$

Which implies

$$d \log \boldsymbol{p} = \Psi \left[\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d \log \boldsymbol{w} - \mathfrak{Q}\mathfrak{T} d \log \theta\right] - d \log \boldsymbol{A}\right]$$

Or equivalently

$$\left(\boldsymbol{I} - \Psi \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \mathcal{L}\right) d \log \boldsymbol{p} = \Psi \left[\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p} - \mathfrak{Q}\mathfrak{T} d \log \theta\right] - d \log \boldsymbol{A}\right]$$

Rearranging yields Proposition 4.

A.2. Proof for Theorem 1 and corrollaries

From household's maximization problem, $p_i c_i = \varepsilon_{c_i}^{\mathcal{D}} G$. Combining with the expression for Domar weights gives:

$$\boldsymbol{\lambda} = \boldsymbol{\Psi}' \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\boldsymbol{D}}.$$

The aggregate labor force, employment, and unemployment are $H^{agg} = \sum_{o=1}^{\mathcal{O}} H_o$, $L^{agg} = \sum_{o=1}^{\mathcal{O}} L_o$, and $U^{agg} = \sum_{o=1}^{\mathcal{O}} U_o$. Log changes in aggregates are given by

$$\begin{aligned} d\log H^{agg} &= \frac{1}{H^{agg}} \boldsymbol{H}' d\log \boldsymbol{H} \\ d\log L^{agg} &= \frac{1}{L^{agg}} \boldsymbol{L}' d\log \boldsymbol{L} \\ d\log U^{agg} &= \frac{1}{U^{agg}} \boldsymbol{U}' d\log \boldsymbol{U} \end{aligned}$$

Substituting in for $d\log \boldsymbol{L}$ gives

$$d\log L^{agg} = \boldsymbol{\Pi}_{L^{agg},A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{L^{agg},H} d\log \boldsymbol{H}$$

Where

$$\begin{aligned} \boldsymbol{\Pi}_{L^{agg},A} &= \frac{1}{L^{agg}} \boldsymbol{L}' \left[\mathcal{L}\boldsymbol{\Pi}_{y,A} - \boldsymbol{\Lambda}_A\right] \\ \boldsymbol{\Pi}_{L^{agg},H} &= \frac{1}{L^{agg}} \boldsymbol{L}' \left[\mathcal{L}\boldsymbol{\Pi}_{y,H} - \boldsymbol{\Lambda}_H\right] \end{aligned}$$

And

$$d\log U^{agg} = \boldsymbol{\Pi}_{U^{agg},A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{U^{agg},H} d\log \boldsymbol{H}$$

Where

$$\begin{aligned} \boldsymbol{\Pi}_{U^{agg},A} &= \frac{1}{U^{agg}} \boldsymbol{U}' \left[\boldsymbol{\Lambda}_A - \mathcal{L}\boldsymbol{\Pi}_{y,A}\right] \\ \boldsymbol{\Pi}_{U^{agg},H} &= \frac{1}{U^{agg}} \boldsymbol{U}' \left[\boldsymbol{I} + \boldsymbol{\Lambda}_H - \mathcal{L}\boldsymbol{\Pi}_{y,H}\right] \end{aligned}$$

A.3. Amplification Proofs

We want to show that $(\boldsymbol{I} - \mathcal{M} - \Xi_\theta)^{-1}$, where $\Xi_\theta = \mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}[\boldsymbol{I} - \mathcal{M}(\boldsymbol{I} + \mathcal{T})]$, is non-negative element-wise.

We have

$$(\boldsymbol{I} - \mathcal{M} - \Xi_\theta)^{-1} = \left(\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1}\right)^{-1} (\boldsymbol{I} - \mathcal{M})^{-1}$$

Since $(\boldsymbol{I} - \mathcal{M})^{-1}$ is a diagonal matrix with positive diagonal elements, it suffices to show that $\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1}$ is an M-matrix, since M-matrices are inverse non-negative.

$\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1}$ is a Z-matrix since its off-diagonals are negative. If $\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1}$ is diagonally dominant then it is also and M-matrix. Let a_{ij} denote the (i, j)-th element of $\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1}$. Row diagonal dominance requires:

$$|a_{ii}| \geq \sum_{j \neq i} |a_{ij}| \quad \forall i$$

For simplicity, we consider the case with one occupation per sector $\mathcal{L} = \boldsymbol{I}$, but the logic behind this proof follows for general $\mathcal{L}$.

To start with, we show that the row sums of $\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is less than or equal to 1.

Let B_{ij} denote the $(i, j) - th$ element of $(\boldsymbol{I} - \boldsymbol{\Omega})$, and Ψ_{ij} the $(i, j) - th$ element of Ψ, and

$$\sum_k \Psi_{ik} B_{kj} = \delta_{ij}$$

We have:

$$\sum_k \Psi_{ik} \sum_j B_{kj} = \sum_k \sum_j \Psi_{ik} B_{kj} = \sum_j \delta_{ij} = 1,$$

since $\boldsymbol{\Psi}$ is the inverse of $(\boldsymbol{I} - \boldsymbol{\Omega})$.

For each k, the (k, k)-th element of $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, β_{kk}, is smaller than or equal to $\sum_j B_{kj}$ by the constant returns of the production functions and non-negative factor shares.

Thus, for each row i in the matrix $\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, the row sum is given by:

$$\sum_k \Psi_{ik} \beta_{kk} \leq 1$$

Let x_{ij} denote the (i, j)-th element of the matrix $\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, row diagonal dominance requires that for each row i,

$$1 - x_{ii}\frac{(1-\eta_i\,(1+\tau_i))}{1-\eta_i} \geq \sum_{j\neq i} x_{ij}\frac{\left(1-\eta_j\left(1+\tau_j\right)\right)}{1-\eta_j}$$

Rewriting it yields:

$$1 \geq \sum_{j} x_{ij}\frac{\left(1-\eta_j\left(1+\tau_j\right)\right)}{1-\eta_j},$$

which holds because $\tau_j \geq 0$.

Since $\tau_j > 0$, we actually have a strict inequality, where

$$1 > \sum_{j} x_{ij}\frac{\left(1-\eta_j\left(1+\tau_j\right)\right)}{1-\eta_j}.$$

Now we can look at the case for general $\mathcal{L}$. We only need to show that the row sum of $\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is no greater than 1.

First, we have the (i, j)-th element of $\mathcal{L}\boldsymbol{\Psi}$ is:

$$\sum_{k=1}^{J} \mathcal{L}_{ik}\Psi_{kj}$$

Thus, the (i, q)-th element of $\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is:

$$\sum_{j=1}^{J} \beta_{jq}\left(\sum_{k=1}^{J} \mathcal{L}_{ik}\Psi_{kj}\right)$$

The row sum for the i-th row of $\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is thus:

$$\sum_{q=1}^{\mathcal{O}}\sum_{j=1}^{J} \beta_{jq}\left(\sum_{k=1}^{J} \mathcal{L}_{ik}\Psi_{kj}\right) = \sum_{k=1}^{J}\mathcal{L}_{ik}\sum_{j=1}^{J}\Psi_{kj}\sum_{q=1}^{\mathcal{O}}\beta_{jq}$$

By definition, for any j, $\sum_{q=1}^{\mathcal{O}} \beta_{jq} \leq \sum_j B_{kj}$, which implies that

$$\sum_{k=1}^{J} \mathcal{L}_{ik} \sum_{j=1}^{J} \Psi_{kj} \sum_{q=1}^{\mathcal{O}} \beta_{jq} \leq \sum_{k=1}^{J} \mathcal{L}_{ik} \leq 1$$

In fact, we can also show that $\boldsymbol{I} - \Xi_\theta \left(\boldsymbol{I} - \mathcal{M}\right)^{-1}$ has value at least 1 on the diagonals. This can be proven by rewriting $\boldsymbol{I} - \Xi_\theta \left(\boldsymbol{I} - \mathcal{M}\right)^{-1}$ as a Neumann series, which converges because

$$1 > \sum_j x_{ij} \frac{\left(1 - \eta_j \left(1 + \tau_j\right)\right)}{1 - \eta_j}.$$

Appendix B. Results for general CRTS production functions and one occupation per sector

In this section we generalize our results to any constant returns to scale production function, under the assumption that there is one type of labor per sector. This generalization results in additional terms that capture how the production elasticities change when shocks hit the economy. The expressions are otherwise similar to above. The model setup is identical, we just do not impose Cobb-Douglas technology and instead impose $\mathcal{O} = J$.

B.1. Price changes

First order changes in prices remain largely unchanged and satisfy

$$\left(\boldsymbol{I} - \Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) d \log \boldsymbol{p} = \boldsymbol{\Psi} \left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left[d \log \boldsymbol{w} - d \log \boldsymbol{p} - \Omega \mathfrak{T} d \log \theta\right] - d \log \boldsymbol{A}\right]$$

B.2. Sales Share Propagation

We can rewrite the goods market clearing condition in terms of Domar weights:

$$y_i = c_i + \sum_{j=1}^{J} x_{ji}$$

$$\Rightarrow \frac{p_i y_i}{\sum_{k=1}^{J} p_k c_k} = \frac{p_i c_i}{\sum_{k=1}^{J} p_k c_k} + \sum_{j=1}^{J} \frac{p_i x_{ji}}{p_j x_j} \frac{p_j x_j}{\sum_{k=1}^{J} p_k c_k}$$

$$\Rightarrow \lambda_i = \varepsilon_{c_i}^{\mathcal{D}} + \sum_{j=1}^{J} \varepsilon_{x_{ji}}^{f_j} \lambda_j, \tag{A1}$$

where $\lambda_i = \frac{p_i y_i}{\sum_{k=1}^{J} p_k c_k}$ is the Domar weight of sector i.

By stacking (A1) for each sector, we get the following expression for Domar weights across the production network.

$$\boldsymbol{\lambda}' = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} + \boldsymbol{\lambda}'\boldsymbol{\Omega}$$

We can see how Domar weights change across the production network by totally differentiating

$$d\boldsymbol{\lambda}' = d\varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} + d\boldsymbol{\lambda}'\boldsymbol{\Omega} + \boldsymbol{\lambda}' d\boldsymbol{\Omega}$$

$$\Rightarrow d\boldsymbol{\lambda}' = \left[d\varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} + \boldsymbol{\lambda}' d\boldsymbol{\Omega} \right] \boldsymbol{\Psi} \tag{A2}$$

The Domar weights will help us express how shocks propagate to output.

B.3. Output changes

We can now write changes in output in terms of changes in tightness, technology, the size of the labor force, and changes in production elasticities, including changes in Domar weights, as

$$d \log \boldsymbol{y} = \boldsymbol{\Psi} \left(d \log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\mathcal{F} + \boldsymbol{\Omega} \mathcal{T}) \, d \log \theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d \log \boldsymbol{H} \right) - \boldsymbol{\Psi} d \log \mathcal{E} + \boldsymbol{\Psi} \left(\operatorname{diag}(\boldsymbol{\Omega} \mathbf{1}) - \boldsymbol{\Omega} \right) d \log \boldsymbol{\lambda}$$

Where $\mathbf{1}$ is a $J \times 1$ vector of ones and $d \log \mathcal{E}$ is the $J \times 1$ vector of diagonal elements of $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d \log \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}'}$.

B.4. Tightness changes

Much like output, changes in tightness now also depends on changes in the elasticities of the production functions. The expressiong for changes in tightness is

$$d \log \theta = \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\Psi d \log \boldsymbol{A} - \left[d \log \boldsymbol{w} - d \log \boldsymbol{p}\right] + \left[\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] d \log \boldsymbol{H}\right] \\ + \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\operatorname{diag}\left(\mathcal{L} d \log \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) + \Psi \left[\left(\operatorname{diag}\left(\boldsymbol{\Omega 1}\right) - \boldsymbol{\Omega}\right) d \log \boldsymbol{\lambda} - d \log \mathcal{E}\right]\right]$$

Where $\Xi_\theta = \Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} [\mathcal{F} + \Omega \mathcal{T}]$. Notice, all terms in the second line are zero assuming Cobb-Douglas production technology.

B.5. Aggregation

Aggregate output now satisfies

$$d \log Y = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} d \log \boldsymbol{c} \\ = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} \left(d \log \varepsilon_{\boldsymbol{c}}^{\mathcal{D}} d \log \boldsymbol{y} - d \log \boldsymbol{\lambda}\right)$$

Appendix C. Data and Calibration Details

This appendix describes the balanced flows formulation of our model and the data in greater detail.

C.1. Balanced Flows Formulation

To bring our model closer to the data, we alter the static setting of our labor markets to a balanced flow setup. This doesn't change the qualitative theoretical results, but allows us to calibrate the model in a data-consistent way. Specifically, this allows us to use unemployment, instead of work force, as the denominator in tightness. Below, we will outline the balanced flow setup and point out how it will alter our propagation and aggregation formulae.

Matching Functions. Now, hires are generated by a constant returns Cobb-Douglas matching function. The number of matches depends on the number of unemployed workers searching for a job in occupation o, U_o, and the number of vacancies posted in occupation o, V_o,

$$h_o = \phi_o U_o^{\eta_o} V_o^{1-\eta_o}.$$

V_o is the sum of sectoral vacancy postings v_{io} and occupational labor market tightness is $\theta_o = \frac{V_o}{U_o}$.

Both the vacancy-filling rate $\mathcal{Q}_o$ and job-finding rate $\mathcal{F}_o$ can be expressed as functions of tightness.

$$\mathcal{Q}_o(\theta_o) = \frac{h_o}{V_o} = \phi_o \theta_o^{-\eta_o}, \quad \mathcal{F}_o(\theta_o) = \frac{h_o}{U_o} = \phi_o \theta_o^{1-\eta_o}.$$

Labor Supply. The balanced flow assumptions says that the number of workers being separated from their jobs each period is equal to the number of unemployed workers finding a job:

$$s \times L_o = \mathcal{F}_o(\theta_o) \times U_o = \mathcal{F}_o(\theta_o) \times (H_o - L_o),$$

where s is an universal separation rate for all workers.

This implies that:

$$\begin{aligned} L_o^s &= \frac{\mathcal{F}_o(\theta_o)}{s + \mathcal{F}_o(\theta_o)} H_o \\ d \log L_o^s &= d \log H_o + \varepsilon_{\theta_o}^{\mathcal{F}_o} d \log \theta_o - \frac{\mathcal{F}_o(\theta_o)}{s + \mathcal{F}_o(\theta_o)} \varepsilon_{\theta_o}^{\mathcal{F}_o} d \log \theta_o \\ &= d \log H_o + \frac{s}{s + \mathcal{F}_o(\theta_o)} \varepsilon_{\theta_o}^{\mathcal{F}_o} d \log \theta_o \\ &= d \log H_o + \frac{s}{s + \mathcal{F}_o(\theta_o)} (1 - \eta_o) d \log \theta_o \end{aligned}$$

Labor Demand. The balanced flow assumption on the labor demand side assumes that the number of jobs being filled equals the number of workers separated from an occupation for each sector:

$$\mathcal{Q}_o(\theta_o) \times V_{io} = s \times L_{io}$$

This implies that the number of recruiters in each sector for each occupation R_{io} is

$$R_{io} = L_{io} - N_{io} = r_o \times V_{io} = \frac{r_o \times V_{io}}{\mathcal{Q}_o(\theta_o)} L_{io} = \frac{r_o \times s}{\mathcal{Q}_o(\theta_o)} (R_{io} + N_{io})$$

Dividing both sides by N_{io} yields the recruiter-producer ratio:

$$\tau_{io}(\theta_o) \equiv \frac{R_{io}}{N_{io}} = \frac{r_o \times s}{\mathcal{Q}_o(\theta_o)} \left(\frac{R_{io}}{N_{io}} + 1\right)$$

Rearranging yields:

$$\tau_{io} = \frac{r_o s}{\mathcal{Q}_o(\theta_o) - r_o s},$$

which is the same for all sectors.

Using this expression for the recruiter producer ratio, total labor demand by sector i for occupation o is

$$L_{io}^d = (1 + \tau_o(\theta_o))N_{io}$$

where N_{io} is determined by the sectors' profit maximization problem.

The aggregate occupation o labor demand is:

$$L_o^d(\theta_o) = \sum_{i=1}^{J} L_{io}^d(\theta_o) = (1 + \tau_o(\theta_o)) \sum_{i=1}^{J} N_{io}$$

From this alternative definition, we have:

$$d\log(1 + \tau_o(\theta_o)) = \frac{\tau_o(\theta_o)}{1 + \tau_o(\theta_o)} \varepsilon_{\theta_o}^{\mathcal{Q}_o} d\log\theta_o,$$

which is the same as the static setup.

Thus, the tightness propagation is

$$d\log\boldsymbol{\theta} = \boldsymbol{\Pi}_{\theta,A} d\log\boldsymbol{A} + \boldsymbol{\Pi}_{\theta,H} d\log\boldsymbol{H}$$

where

$$\begin{aligned}
\boldsymbol{\Pi}_{\theta,A} &= \left[\boldsymbol{u}\left(\boldsymbol{I} - \mathcal{M}\right) - \Xi_\theta\right]^{-1} \left(\mathcal{L}\boldsymbol{\Psi} - \boldsymbol{\Lambda}_A\right), \\
\boldsymbol{\Pi}_{\theta,H} &= \left[\boldsymbol{u}\left(\boldsymbol{I} - \mathcal{M}\right) - \Xi_\theta\right]^{-1} \left(\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \boldsymbol{\Lambda}_H\right), \\
\Xi_\theta &= \mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left[\boldsymbol{u} - \mathcal{M}\left(\boldsymbol{u} + \mathcal{T}\right)\right].
\end{aligned}$$

and the output propagation is:

$$d\log \boldsymbol{y} = \Pi_{y,A} d\log \boldsymbol{A} + \Pi_{y,H} d\log \boldsymbol{H},$$

where

$$\Pi_{y,A} = \underbrace{\Psi}_{\text{frictionless}} + \underbrace{\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{u} - \underbrace{\mathcal{M}(\boldsymbol{u} + \mathcal{T})}_{\text{search cost}} \right) \Pi_{\theta,A}}_{\text{tightness adjustment}},$$

$$\Pi_{y,H} = \underbrace{\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}}_{\text{frictionless}} + \underbrace{\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\boldsymbol{u} - \mathcal{M}(\boldsymbol{u} + \mathcal{T})) \Pi_{\theta,H}}_{\text{tightness adjustment}}$$

C.2. Dampening under tight labor market

Here, we illustrate how tight labor markets can dampen the impact of positive shocks.

When labor markets are tight, $\boldsymbol{u} - \mathcal{M}(\boldsymbol{u} + \mathcal{T}) < 0$ element-wise. To show that the search channel $\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{u} - \underbrace{\mathcal{M}(\boldsymbol{u} + \mathcal{T})}_{\text{search cost}} \right) \Pi_{\theta,A} < 0$ element-wise, we only need to show that $\Pi_{\theta,A} < 0$ element-wise.

Note that we have

$$\Pi_{\theta,A} = \left[\boldsymbol{u}(\boldsymbol{I} - \mathcal{M}) - \Xi_\theta\right]^{-1} (\mathcal{L}\Psi - \Lambda_A),$$
$$\Xi_\theta = \mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} [\boldsymbol{u} - \mathcal{M}(\boldsymbol{u} + \mathcal{T})].$$

For simplicity, assume wages adjust proportionally to the network-adjusted productivity gain:

$$\Lambda_{\boldsymbol{A}} = \delta \mathcal{L}\Psi$$

for a given $0 \leq \delta < 1$.

Therefore,

$$\Pi_{\theta,A} = (1 - \delta) \left[\boldsymbol{u}(\boldsymbol{I} - \mathcal{M}) - \Xi_\theta\right]^{-1} (\mathcal{L}\Psi)$$

Further, we assume that $\mathcal{L} = \boldsymbol{I}$, which means sectors only employ sector-specific workers. This gives us:

$$\begin{aligned}\Pi_{\theta,A} &= (1-\delta)\left[\boldsymbol{u}(\boldsymbol{I}-\mathcal{M}) - \Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left[\boldsymbol{u}-\mathcal{M}(\boldsymbol{u}+\mathcal{T})\right]\right]^{-1}(\Psi)\\ &= (1-\delta)\left[\boldsymbol{u}(\boldsymbol{I}-\mathcal{M})(\boldsymbol{I}-\boldsymbol{\Omega}) - \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left[\boldsymbol{u}-\mathcal{M}(\boldsymbol{u}+\mathcal{T})\right]\right]^{-1}\end{aligned}$$

Note that because $\boldsymbol{I}-\mathcal{M}$ is strictly diagonally dominant and $\boldsymbol{u}(\boldsymbol{I}-\mathcal{M})$ is a positive diagonal matrix, $\boldsymbol{u}(\boldsymbol{I}-\mathcal{M})(\boldsymbol{I}-\boldsymbol{\Omega})$. In addition, $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is a positive diagonal matrix and $\boldsymbol{u}-\mathcal{M}(\boldsymbol{u}+\mathcal{T}) < 0$ element-wise. This implies that $\boldsymbol{u}(\boldsymbol{I}-\mathcal{M})(\boldsymbol{I}-\boldsymbol{\Omega}) - \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left[\boldsymbol{u}-\mathcal{M}(\boldsymbol{u}+\mathcal{T})\right]$ is strictly diagonally dominant, and $\Pi_{\theta,A} > 0$ element-wise. Therefore, the search channel dampens the impact of productivity shocks.

C.3. Input-output matrix

We use the 3-digit 2021 BEA Make and Use tables accessible at https://www.bea.gov/industry/input-output-accounts-data to calculate the relevant production elasticities[11]. The 3-digit Make and Use tables record the nominal amount of each 71 commodities made by and used by each of 71 industries. The commodities are denoted using the same codes as the industries, but they are conceptually distinct as each industry can produce more than one commodity.

For consistency with the industry classifications in JOLTs and the CPS unemployment by sector series, we collapse the 3-digit tables to a 13 sector table. The table below outlines the mapping from the NAICS 2-digit classification codes to our industry classifications. The mapping from 2-digit codes to 3-digit codes is readily available online.

[11]See https://www.bea.gov/sites/default/files/methodologies/IOmanual_092906.pdf for a detailed description of how these tables are generated.

Industry Name	Short Name	2-digit codes
Leisure and Hospitality	accom	71, 72
Construction	const	33
Durable goods	dur	33DG
Education and Health Services	edhealth	61, 62
Financial Activities	fin	52, 53
Government	gov	G
Information	info	51
Mining	mining	21
Nondurable good	nondur	11, 31ND
Other services, except government	other	81
Professional and business services	profserv	54, 55, 56
Wholesale and Retail trade	trade	42, 44RT
Transportation and Utilities	trans	22, 48TW

TABLE A1. Mapping from NAICS classification to our industries.

With the 13-sector make and use tables in hand, we can construct production elasticities in intermediate inputs and to labor, and demand elasticities. Let M_{ij} denote the nominal value of commodity i made by industry j. Let U_{ij} denote the nominal amount of commodity i used by industry j. The two tables below demonstrate the elements of the Make and Use tables.

	Sector 1	Sector 2	$\cdots$	Sector J	Total Industry Output
Sector 1	M_{11}	M_{21}	$\cdots$	M_{J1}	$\sum_{i=1}^{J} M_{i1}$
Sector 2	M_{12}	M_{22}	$\cdots$	M_{J2}	$\sum_{i=1}^{J} M_{i2}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Sector J	M_{1J}	M_{2J}	$\cdots$	M_{JJ}	$\sum_{i=1}^{J} M_{iJ}$
Total Commodity Output	$\sum_{j=1}^{J} M_{1j}$	$\sum_{j=1}^{J} M_{2j}$	$\cdots$	$\sum_{j=1}^{J} M_{Jj}$	—

TABLE A2. Make table

	Sector 1	Sector 2	$\cdots$	Sector J	Total Intermediate Uses	Total Final Uses
Sector 1	U_{11}	U_{12}	$\cdots$	U_{1J}	$\sum_{j=1}^{J} U_{1j}$	$\sum_{j=1}^{J} U_{1j} + p_1 c_1$
Sector 2	U_{21}	U_{22}	$\cdots$	U_{2J}	$\sum_{j=1}^{J} U_{2j}$	$\sum_{j=1}^{J} U_{2j} + p_2 c_2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
Sector J	U_{J1}	U_{J2}	$\cdots$	U_{JJ}	$\sum_{j=1}^{J} U_{Jj}$	$\sum_{j=1}^{J} U_{Jj} + p_J c_J$
Total Intermediate Inputs	$\sum_{i=1}^{J} U_{i1}$	$\sum_{i=1}^{J} U_{i2}$	$\cdots$	$\sum_{i=1}^{J} U_{iJ}$	—	—
Total industry output	$\sum_{i=1}^{J} U_{i1} + w_1(1+\tau_1)N_1$	$\sum_{i=1}^{J} U_{i2} + w_2(1+\tau_2)N_2$	$\cdots$	$\sum_{i=1}^{J} U_{iJ} + w_J(1+\tau_J)N_J$	—	—

TABLE A3. Use table

First, we calculate the fraction of commodity i produced by industry j by dividing the entry in along each row by the corresponding "total industry output"

$$m_{ij} = \frac{M_{ij}}{\sum_{j=1}^{J} M_{ji}}$$

Second, we calculate the share of commodity i in industry j's total uses as by dividing each entry in the column corresponding to industry j by the corresponding "Total industry output"

$$u_{ij} = \frac{U_{ij}}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

We form the two matrices

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{21} & \cdots & m_{J1} \\ m_{12} & m_{22} & \cdots & m_{J2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{1J} & m_{2J} & \cdots & m_{JJ} \end{bmatrix}, \boldsymbol{U} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1J} \\ u_{21} & u_{22} & \cdots & u_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ u_{J1} & u_{J2} & \cdots & u_{JJ} \end{bmatrix}$$

Then, we can calculate our input output matrix by

$$\boldsymbol{\Omega} = [\boldsymbol{MU}]'$$

Given our assumption of constant returns to scale and zero profits, the difference between total intermediate inputs and total industry output is the nominal income paid to workers in each sector. We abstract from the other components of total industry output in the IO accounts, taxes and gross operating surplus, as they have no model counterpart in our setup. We can therefore calculate the labor elasticities from the Use table as "Total industry output" – "Total intermediate inputs" ÷ "Total industry output."

$$\varepsilon^{f_j}_{N_j} = \frac{w_j(1+\tau_j)N_j}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

Finally, we can back out the demand elasticities from "Total intermediate uses" and "Total final uses" columns of the Uses table.

$$p_i c_i = \sum_{j=1}^{J} U_{ij} + p_i c_i - \sum_{j=1}^{J} U_{ij}$$

We can the work out the elasticities by

$$\varepsilon^{\mathcal{D}}_{c_i} = \frac{p_i c_i}{\sum_{i=1}^{J} p_i c_i}$$

Finally, to ensure that constant returns to scale holds we rescale our elasticities proportionally to ensure they sum to one. This adjustment is minor and is only needed because we drop the small "Used" and "rest of world adjustment" categories. It does not change any elasticity by more than 3 percent.

We report the resulting estimates of the production elasticities, labor elasticities, and demand elasticities in the tables below. In tables A4 and A5 we assume all non-intermediate, non-energy, and non-capital, spending goes to labor income, which automatically imposes constant returns but leads to large labor shares.

Sector	Labor Elasticity $\left(\varepsilon_N^f\right)$	Demand Elasticity $\left(\varepsilon_c^{\mathcal{D}}\right)$
accom	0.510	0.051
const	0.474	0.065
dur	0.434	0.138
edhealth	0.616	0.129
fin	0.617	0.165
gov	0.626	0.132
info	0.571	0.043
mining	0.518	0.008
nondur	0.352	0.151
other	0.608	0.024
profserv	0.591	0.071
trade	0.523	0.000
trans	0.492	0.022

TABLE A4. Labor elasticities and demand elasticities according the BEA make use tables for 13-industry classification, rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

	accom	const	dur	edhealth	fin	gov	info	mining	nondur	other	profserv	trade	trans
accom	0.029	0.002	0.017	0.003	0.099	0.007	0.036	0.008	0.103	0.014	0.143	0.000	0.030
const	0.001	0.000	0.282	0.000	0.033	0.001	0.016	0.022	0.089	0.006	0.072	0.000	0.004
dur	0.001	0.001	0.393	0.000	0.016	0.001	0.012	0.013	0.056	0.003	0.055	0.004	0.012
edhealth	0.019	0.000	0.030	0.015	0.103	0.006	0.028	0.004	0.056	0.009	0.101	0.000	0.012
fin	0.011	0.023	0.007	0.000	0.195	0.005	0.020	0.001	0.008	0.005	0.083	0.001	0.022
gov	0.006	0.023	0.048	0.011	0.055	0.004	0.032	0.013	0.081	0.012	0.072	0.000	0.017
info	0.017	0.001	0.040	0.000	0.047	0.003	0.125	0.002	0.015	0.005	0.160	0.001	0.013
mining	0.001	0.005	0.010	0.000	0.060	0.002	0.017	0.128	0.056	0.001	0.090	0.000	0.022
nondur	0.001	0.003	0.044	0.000	0.020	0.002	0.008	0.124	0.377	0.003	0.040	0.004	0.021
other	0.013	0.005	0.066	0.010	0.113	0.005	0.033	0.003	0.032	0.013	0.086	0.000	0.012
nprofserv	0.020	0.000	0.029	0.001	0.070	0.004	0.054	0.002	0.026	0.008	0.176	0.000	0.017
trade	0.006	0.002	0.025	0.002	0.104	0.009	0.038	0.002	0.031	0.015	0.159	0.021	0.063
trans	0.014	0.007	0.022	0.000	0.080	0.015	0.025	0.030	0.080	0.011	0.093	0.001	0.129

TABLE A5. Production elasticities to intermediate inputs at 13-sector level ($\mathbf{\Omega}$), rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

C.4. Matching Parameters

We estimate the parameters of the sector specific matching function from monthly data on hires and vacancies from JOLTs and unemployment from the CPS. In particular, we estimate

$$\log Hires_{i,t} = \alpha_i + \eta_i \log U_{i,t} + (1 - \eta_i) \log V_{i,t} + \epsilon_{i,t}$$

by least squares. $\phi_{i,t} = \exp(\alpha_i + \epsilon_{i,t})$ is the time varying matching efficiency in sector i and η_i is the matching elasticity with respect to unemployment in sector i. Effectively, we are choosing η_i to minimize the unexplained component of matching efficiency $\epsilon_{i,t}$. We report the resulting estimates in the table below.

	$\exp(\widehat{\alpha}_i)$	Unemployment Elasticity $(\widehat{\eta}_i)$
accom	1.185	0.401
const	1.106	0.507
dur	0.688	0.364
edhealth	0.703	0.336
fin	0.705	0.329
gov	0.640	0.291
info	0.703	0.275
mining	1.236	0.262
nondur	0.779	0.391
other	0.848	0.441
profserv	1.077	0.372
trade	1.009	0.430
trans	0.862	0.439

TABLE A6. Matching function parameter estimates. Based on monthly hiring, unemployment, and vacancy data from Jan 2000 to Feb 2023.

Finally, we use the sector level proportion of HR workers as a proxy for the recruiter producer ratio. The resulting recruiter producer ratios are reported below

	τ_i
accom	0.002
const	0.002
dur	0.007
edhealth	0.005
fin	0.008
gov	0.011
info	0.013
mining	0.005
nondur	0.007
other	0.018
profserv	0.020
trade	0.003
trans	0.001

TABLE A7. Estimated recruiter producer ratios based on the number of HR workers in industry i over total employment in industry i.

C.5. Computing Occupational Worker Share

For our occupational labor market calibration, we need to compute $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, which is the occupational worker elasticity of production. To do this, we obtain wage and employment data for ONET major occupations at 3-digit sector level from the Occupational Employment and Wage Statistics (OES). For each sector i, we compute $\varepsilon_{N_i o}^{f}$ as:

$$\varepsilon_{N_{io}}^{f} = \varepsilon_{Ni}^{f} \frac{w_{io} L_{io}}{\sum_o w_{io} L_{io}},$$

where ε_{N}^{f} is the labor share we obtained earlier from the input-output table.

Table A8 contains our calibration estimates.

	Admin	Agg	Arts	Bus Ops	Care	Clean	Cons	Educ	Eng	Food S	Health P	Health S	Legal	Manag	Math	Prod	Prot S	Repair	Sales	Science	Soc S	Trans
accom	1.7	0.0	0.9	0.8	1.9	1.6	0.0	0.2	0.0	23.9	0.1	0.0	0.0	3.5	0.1	0.3	0.4	0.7	1.4	0.0	0.0	0.8
const	2.8	0.0	0.1	3.0	0.0	0.1	22.9	0.0	0.7	0.0	0.0	0.0	0.0	5.6	0.1	0.6	0.0	3.4	1.0	0.1	0.0	1.1
dur	1.9	0.0	0.2	2.3	0.0	0.1	0.6	0.0	4.0	0.0	0.0	0.0	0.1	4.6	1.9	11.5	0.0	1.3	1.2	0.2	0.0	1.4
edhealth	4.1	0.0	0.4	1.4	0.6	0.8	0.1	12.9	0.0	0.7	17.8	5.3	0.0	4.3	0.7	0.1	0.3	0.4	0.1	0.6	2.1	0.4
fin	7.2	0.0	0.2	10.6	0.1	0.3	0.1	0.0	0.1	0.0	0.3	0.0	0.6	8.4	3.2	0.0	0.1	1.3	6.3	0.0	0.1	0.3
gov	6.8	0.1	0.5	8.6	0.6	0.8	2.3	1.0	2.6	0.2	4.0	0.5	2.7	6.6	2.4	0.9	12.0	2.3	0.2	2.5	2.9	2.1
info	2.4	0.0	4.4	4.0	0.2	0.0	0.1	0.1	0.6	0.1	0.0	0.0	0.3	7.2	10.3	0.1	0.0	2.0	3.2	0.0	0.0	0.2
mining	1.3	0.0	0.0	1.4	0.0	0.0	7.3	0.0	1.8	0.0	0.0	0.0	0.1	3.7	0.4	1.4	0.0	1.9	0.6	0.7	0.0	2.4
nondur	1.8	0.1	0.2	1.2	0.0	0.2	0.1	0.0	1.0	0.3	0.0	0.0	0.0	3.3	0.3	9.8	0.0	1.7	1.2	0.8	0.0	2.3
other	6.4	0.0	1.7	6.1	8.0	0.8	0.2	0.8	0.3	0.5	0.3	0.7	0.3	9.4	1.0	2.4	0.3	9.5	1.9	0.3	1.6	3.4
profserv	5.4	0.0	1.1	8.6	0.1	2.3	0.7	0.1	3.4	0.1	1.2	0.3	3.1	10.5	7.9	1.0	1.1	0.7	2.2	1.2	0.1	1.7
trade	3.9	0.1	0.5	1.7	0.1	0.2	0.1	0.0	0.2	0.7	1.6	0.1	0.0	5.0	0.8	1.1	0.1	2.3	15.2	0.1	0.0	6.0
trans	6.2	0.0	0.0	0.4	0.0	0.2	0.4	0.0	0.4	0.0	0.0	0.0	0.0	0.5	0.2	0.9	0.0	2.9	0.5	0.1	0.0	18.2

TABLE A8. Occupational worker elasticity of output, in percentage terms, rounded to 1 decimal place.

C.6. Imputing Occupation Labor Market Parameters

For the occupational labor market specification, we need to calibrate unemployment, vacancy, and tightness of each occupation. We currently don't have access to occupational labor market characteristics, so we instead impute these parameters. For simplicity, we assume the total number of unemployment and vacancy for an occupation is the sum of unemployment and vacancy across sectors, weighted by the sectors' wage shares of that particular occupation:

$$V_o = \sum_i \frac{\varepsilon^f_{N_{io}}}{\varepsilon^f_{N_i}} V_i,$$

$$U_o = \sum_i \frac{\varepsilon^f_{N_{io}}}{\varepsilon^f_{N_i}} U_i,$$

where V denotes vacancy and U denotes unemployment. The intuition behind this is that each sector's contribution to vacancy postings and the number of people looking for jobs in that sector for an occupation is proportional to how much the sector relies on that occupation.

Note that, with this imperfect simplifying assumption, we can back out changes in sectoral tightness. For sector j, the first-order response in tightness is: Tightness for sector j is:

$$\begin{aligned}
\theta_j = \frac{V_j}{U_j} &= \frac{\sum_o \frac{V_{jo}}{V_o} V_o}{\sum_o \frac{U_{jo}}{U_o} U_o} \\
\Rightarrow d\log\theta_j &= d\log V_j - d\log U_j \\
&= \sum_o \frac{V_o}{V_j}\frac{V_{jo}}{V_o} d\log V_o - \frac{U_o}{U_j}\frac{U_{jo}}{U_o} d\log U_o \\
&= \sum_o \frac{V_{jo}}{V_j} d\log V_o - \frac{U_{jo}}{U_j} d\log U_o \\
&= \sum_o \frac{\varepsilon^f_{N_{jo}}}{\varepsilon^f_{N_j}} d\log\theta_o.
\end{aligned}$$

We estimate the matching elasticities using the same methodology from appendix section C.4. Table A9 reports the estimated coefficients.

Occupation	$\widehat{\alpha}_i$	Unemployment Elasticity ($\hat{\eta}_o$)
Admin	0.883	0.368
Agg	0.907	0.390
Arts	0.939	0.352
Bus Ops	0.879	0.356
Care	0.978	0.378
Clean	1.009	0.372
Cons	1.054	0.461
Educ	0.713	0.338
Eng	0.871	0.357
Food S	1.163	0.398
Health P	0.749	0.346
Health S	0.731	0.343
Legal	0.923	0.354
Manag	0.915	0.368
Math	0.905	0.342
Prod	0.797	0.374
Prot S	0.744	0.319
Repair	0.909	0.390
Sales	0.975	0.398
Science	0.829	0.355
Soc S	0.701	0.334
Trans	0.924	0.406

TABLE A9. Matching parameters for major occupations

Additionally, following our vacancy assumption, we assume the number of recruiters each sector dedicates to recruiting a particular occupation is proportional to the occupation elasticity of production. In other words:

$$R_o = \sum_i \frac{\varepsilon^f_{N_{io}}}{\varepsilon^f_{N_i}} R_i,$$

Occupation	τ_o
Admin	0.006
Agg	0.002
Arts	0.011
Bus Ops	0.014
Care	0.006
Clean	0.006
Cons	0.004
Educ	0.005
Eng	0.017
Food S	0.002
Health P	0.009
Health S	0.003
Legal	0.027
Manag	0.019
Math	0.020
Prod	0.006
Prot S	0.009
Repair	0.007
Sales	0.005
Science	0.014
Soc S	0.006
Trans	0.003

TABLE A10. Recruiter producer ratios based on the number of estimated recruiters in occupation o.

where R_i is the number of recruiters in sector i. This is also implicitly assuming that the recruiting cost for the occupations are the same.

Since we have the total employment for each occupation from the OES, we can therefore compute the recruiter-producer ratios. Table A10 reports the estimated recruiter-producer ratio for different occupations.

Appendix D. Robustness of calibration results to alternative wage assumptions

D.1. Robustness to Alternative Wage Assumptions

Because wages play an important role in how shocks propagate in our model economy, in this section we check the robustness of our quantitative results to alternative assumptions about wages. Figures A1A and A1B plot the response of output and the unemployment rate to a 1% positive shock to durable manufacturing productivity under the following wage assumptions:

(i) The blue bars labeled "Rigid Real" assume that network-price-adjusted wages do not change in response to the shock.

$$\text{(Rigid Real)} \qquad d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = 0.$$

(ii) The green bars labeled "0.9MP" assume that network-price-adjusted wages change by 0.9 of the change in the marginal product of productive labor.

$$\text{(0.9MP)} \qquad d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = 0.9 d\log \boldsymbol{MP}.$$

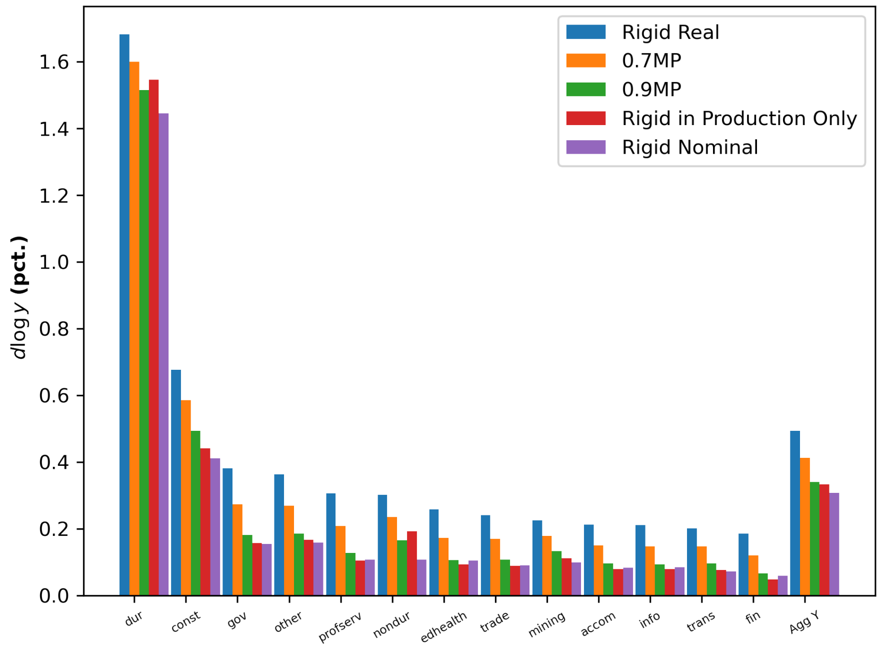
(iii) The red bars labeled "Rigid in Production Only" assume that wages change by exactly as much as the marginal product of labor in all occupations except for production workers. We assume that network-price-adjusted wage changes are zero for production workers. Therefore, this specification imposes rigid wages in the production occupation but flexible wages in every other occupation.

(iv) The purple bars labeled "Rigid Nominal" assume that the nominal wage does not change.

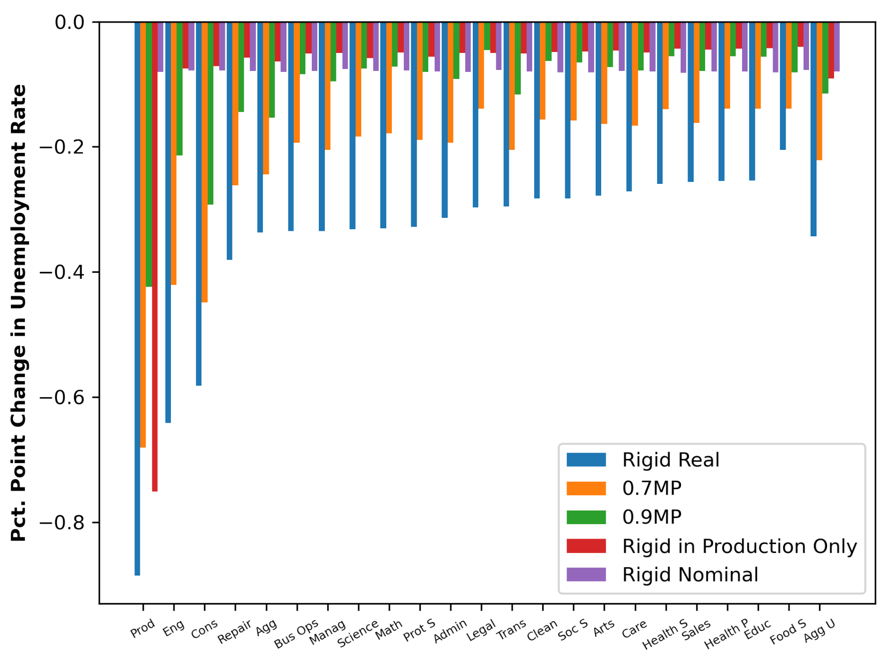
$$\text{(Rigid Nominal)} \qquad d\log \boldsymbol{w} = 0 \text{ or } d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = -\mathcal{L} d\log \boldsymbol{p}.$$

As Corollary 2 shows, Hulten's theorem holds under the MP wage assumption, making the purple bars in Figures A1A and A1B identical to the orange bars that represent production linkages only. Additionally, there are no changes in the unemployment rate when wage changes are proportional to changes in the marginal product of labor. However, the results in Section 4.4 are quantitatively robust to the other three wage assumptions we test. Combining production linkages with labor market frictions leads to non-trivial amplification of output response and widespread effects on unemployment, even when wages move by nearly as much as the marginal product of labor, as demonstrated by the orange bars. In fact, the green bars show that wage rigidity in just a single occupation is enough to generate amplification of the response of output and large localized changes in unemployment.

The amplification effect diminishes with rigid nominal wages since relative price changes across sectors lead to corresponding wage changes, partially mitigating labor market frictions. Under this wage specification, changes in unemployment are also smaller but remain non-trivial and are widespread across all occupations.



A. Sectoral and aggregate output response



B. Unemployment rate response

FIGURE A1. Robustness to alternative wage assumptions: Response of output and the unemployment rate to a 1% shock to technology in the durable manufacturing sector.